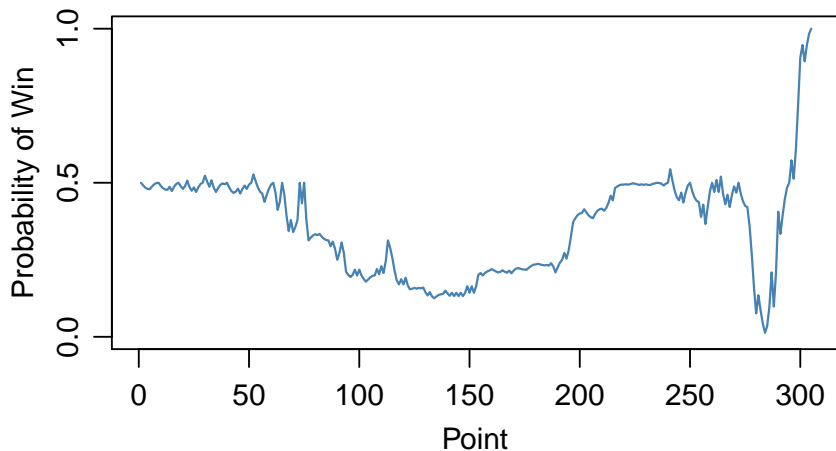


Suspense and Surprise

Jeffrey Ely Alexander Frankel Emir Kamenica

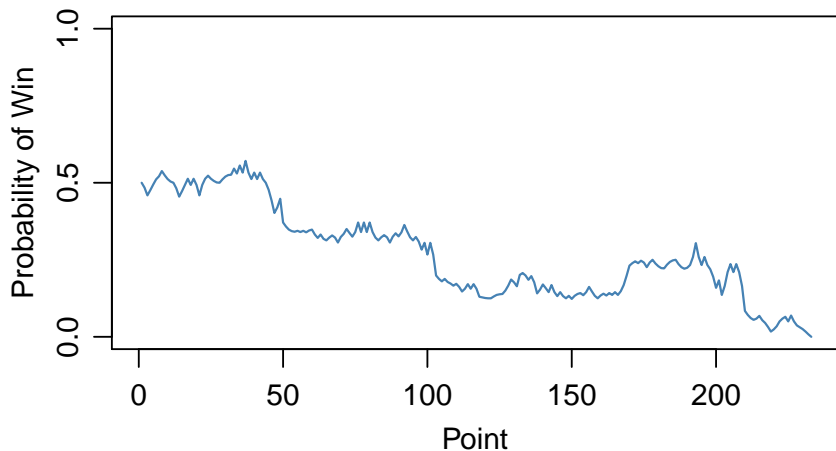
July 24, 2012

2011 US Open Semi-Finals



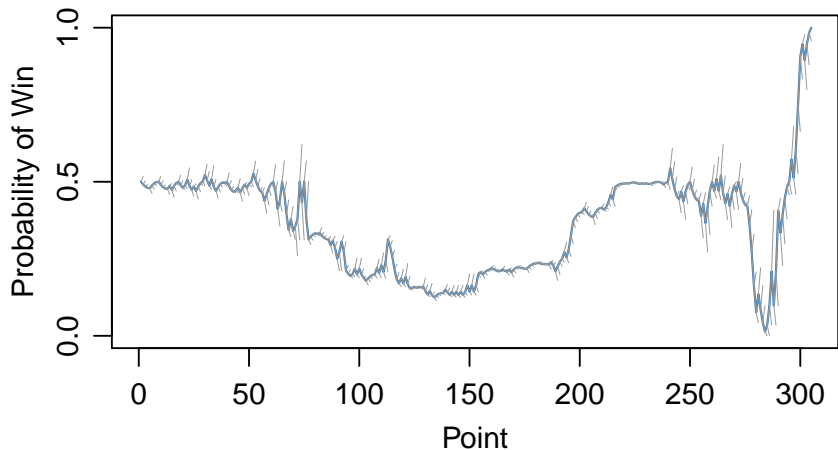
The dynamics of the probability that Djoković beats Federer

2011 US Open Semi-Finals



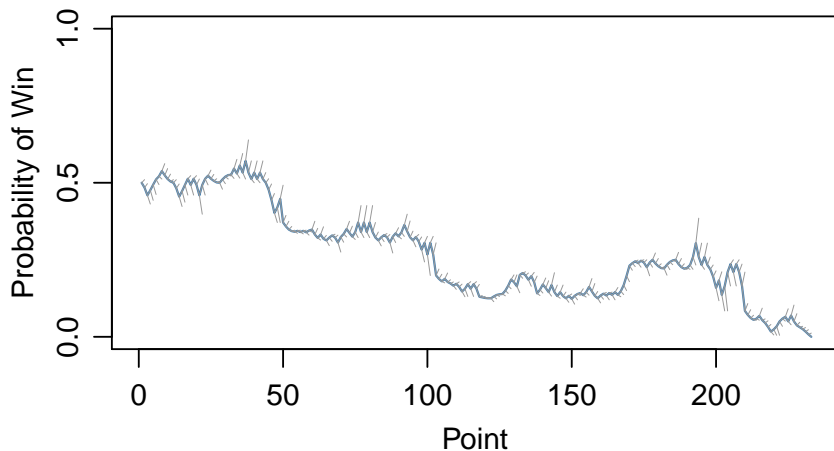
The probability that Murray beats Nadal.

2011 US Open Semi-Finals



The tendrils represent events that could have happened.

2011 US Open Semi-Finals



The tendrils represent events that could have happened.

Suspense and Surprise

Belief processes can be used to model drama, an important source of entertainment.

- We formalize preferences that capture two dimensions of drama:
 - ▶ Surprise: realized movements in beliefs.
 - ▶ Suspense: variance in the distribution of beliefs.
- We model entertainment as an *information policy* and study its optimal design.
 - ▶ The unconstrained optimum, e.g. writing a mystery novel.
 - ▶ Constrained problems, e.g. how to seed a tournament, how many games in a playoff series.

Framework

- There is a finite set of states.

$$\omega \in \Omega = \{0, 1, \dots, n\}$$

- The time horizon is finite.

$$t \in \{0, 1, \dots, T\}$$

- The agent (audience, spectator, reader, etc.) has a belief at each time t .

$$\mu_t = (\mu_t^0, \dots, \mu_t^n) \in \Delta(\Omega)$$

Information Revelation

- The principal (producer, novelist, etc.) reveals a signal in each period

$$\pi : \Omega \rightarrow \Delta(S)$$

where S is a finite set of outcomes.

- An *information policy* is the principal's strategy for choosing signals after each history:

$$\tilde{\pi} : \{0, 1, \dots, T\} \times \Delta(\Omega) \rightarrow \Pi.$$

where Π is the set of all signals.

Bayesian Agent

The main constraint on the principal is that the agent cannot be fooled: He correctly interprets the signal and appropriately updates his belief about the state.

The Information Policy Induces a Stochastic Process for Beliefs

- Given a current belief, μ_t , the signal $\tilde{\pi}(t, \mu_t)$ induces a lottery over period $t + 1$ beliefs

$$\tilde{\mu}_{t+1} \in \Delta(\Delta(\Omega))$$

- given a prior μ_0 , an information policy induces a *belief martingale*

$$\tilde{\mu} = (\tilde{\mu}_t)_{t=0}^T$$

where

- ▶ $\tilde{\mu}_0$ assigns probability 1 to μ_0
- ▶ The *belief path* $(\mu_t)_{t=0}^T$ is the realization of $\tilde{\mu}$,
- ▶ $E_t [\tilde{\mu}_{t+1}] = \mu_t$

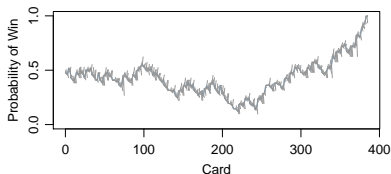
Going In The Other Direction

Lemma (Kamenica-Gentzkow (2011))

For every belief martingale there is an information policy that generates it.

Why Do People Spend *Time* in Casinos?

- Economists and psychologists appeal to non-standard preferences to explain the taste for gambling.
- However, what is left unexplained is why gamblers prefer to experience their gains and losses *over time*.
- An evening of gambling is entertainment, indeed *experiencing* the swings of fortune is the entertainment.



Preference for Surprise

We model a preference for surprise via the following utility function

$$U_{surp} = \sum_{t=1}^T u \left(\sum_{\omega} (\mu_t^{\omega} - \mu_{t-1}^{\omega})^2 \right)$$

where u is increasing and concave.

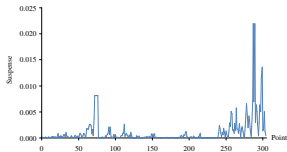
Preference for Suspense

We model a preference for suspense via the following utility function

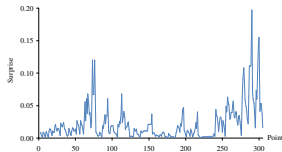
$$U_{susp} = \sum_{t=0}^{T-1} u \left(E_t \sum_{\omega} (\tilde{\mu}_{t+1}^{\omega} - \mu_t^{\omega})^2 \right)$$

where u is increasing and concave.

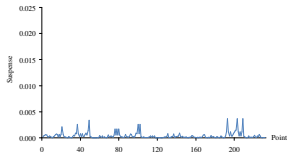
Comparing the Two Semi-Finals



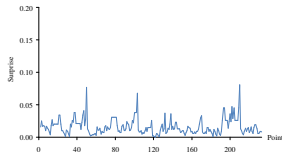
(a) Suspense in Djoković-Federer



(b) Surprise in Djoković-Federer



(c) Suspense in Murray-Nadal



(d) Surprise in Murray-Nadal

Rooting For The Underdog

Spectators who root for the underdog, or for a comeback, reveal a preference for surprise and suspense.

- It is surprising when the underdog wins or when the trailing team comes from behind.
- When the currently trailing team scores, the remainder of the match will have more surprise and suspense.

Maximizing Suspense and Surprise

How do you design an information policy to maximize drama?

Optimal Information Policies

In the reality television show *American Idol*, the host plays up the drama of announcing the loser.

- He brings two contestants to the front.
- He calls out the name of one of them.
- Then he says either “you are out” or “you are safe”

Optimally used, this strategy can increase surprise and suspense relative to a single-stage announcement.

Optimal Information Policies

- Given μ_0 and T
- The principal chooses a martingale $\tilde{\mu}$ to maximize suspense

$$E_{\tilde{\mu}} \sum_{t=0}^{T-1} u \left(E_t \sum_{\omega} (\tilde{\mu}_{t+1}^{\omega} - \mu_t^{\omega})^2 \right)$$

or surprise

$$E_{\tilde{\mu}} \sum_{t=1}^T u \left(\sum_{\omega} (\mu_t^{\omega} - \mu_{t-1}^{\omega})^2 \right)$$

- (We solve these problems separately.)

Interpretations

- Novels.

- ▶ An author (the principal) churns out novels according to an information policy generating a martingale $\tilde{\mu}$.
- ▶ Each new realization from $\tilde{\mu}$ is another novel.
- ▶ The reader knows the martingale and reads the novel, experiencing its suspense/surprise.
- ▶ For example, mystery novels.

- Sports

- ▶ The rule-setting body (the principal) determines the information policy.
- ▶ Each match is a realization from the corresponding martingale.
- ▶ Spectators know the rules and the relative strengths of the two teams.
- ▶ They watch the match to experience its suspense/surprise.

The M. Night Shyamalan Dilemma

- There are information policies with dramatic “plot twists.”
 - ▶ After a history in which beliefs attach high probability to state ω
 - ▶ A signal which causes the probability of state ω' to jump.
- Paths with many plot twists give the highest payoffs *ex post*.
- Without commitment power, the principal is tempted to follow these maximally surprising paths.
- But then they lose their surprise value.
- This is the M. Night Shyamalan Dilemma.

Commitment

- We are assuming the principal has commitment power.
- Thus, more dramatic plot twists are necessarily less likely to occur.
- It is in this sense that the “rationality” of the agent is the primary constraint on the principal.
- ESPN Classic.
- Suspense versus Surprise.

Suspense-Optimum: Characterization

- The prior is μ_0 .
- Suppose that the principal reveals the state in a single period.
- The variance of the updated belief is

$$\sum_{\omega=0}^n (\tilde{\mu}_1^{\omega} - \mu_0^{\omega})^2$$

Suspense-Optimum: Characterization

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$$\sum_{\omega=0}^n \mu_0^{\omega} (1 - \mu_0^{\omega})$$

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Suspense-Optimum: Characterization

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- It is optimal to fully reveal the state eventually but
- The more interesting information policies are gradually revealing, with variance σ_t^2 of date $t + 1$ beliefs.

Budget of Variance

Define $\Psi(\mu) = \sum_{\omega=0}^n \mu^\omega (1 - \mu^\omega)$.

Lemma

Every information policy generates the same total variance

$$\sum_{t=0}^{T-1} \sigma_t^2 = \Psi(\mu_0).$$

- Define the random variable $\tilde{\mu}_t - \mu_{t-1}$, the change in beliefs.
- Its variance is the same as σ_t^2 because its expectation is zero, i.e. $\tilde{\mu}_t$ is a martingale.
- This new sequence of random variables is uncorrelated across t because $\tilde{\mu}_t$ is a martingale.
- The sum of the variances of a sequence of uncorrelated random variables is equal to the variance of the sum.
- The sum of these random variables corresponds to revelation in a single period.

Suspense-Optimum Reformulated

To maximize suspense the principal solves

$$\begin{aligned} \max \quad & \sum_{t=0}^{T-1} u(\sigma_t^2) \\ \text{s.t.} \quad & \sum_{t=0}^{T-1} \sigma_t^2 = \Psi(\mu_0) \end{aligned}$$

Because $u(\cdot)$ is concave, it is optimal to smooth variance across periods to the extent possible.

Suspense Optimum

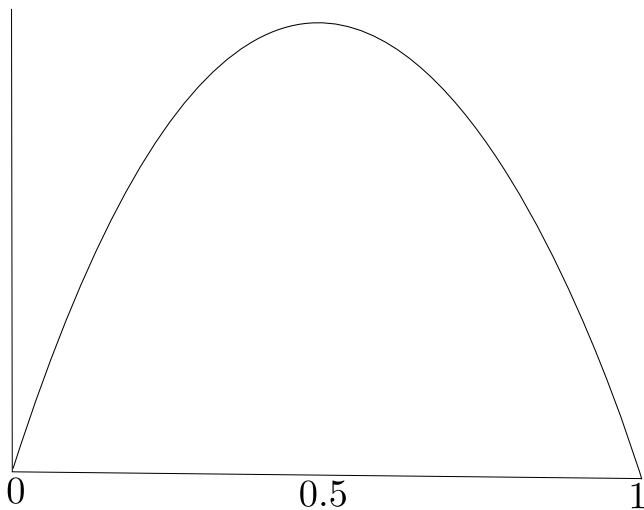
Proposition

There exists a martingale with constant variance, i.e. σ_t^2 constant across t .

- This martingale is therefore suspense-optimal and any suspense-optimal policy must have constant variance.
- A constant variance martingale “consumes” an equal portion of the initial budget $\Psi(\mu_0)$ each period.
- After t periods, the *residual variance* must be $(\frac{T-t}{T})\Psi(\mu_0)$.
- That is, the posterior μ_t must satisfy

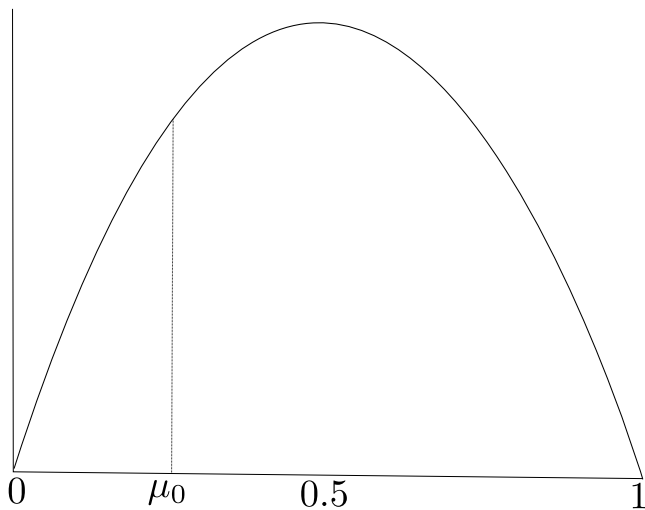
$$\Psi(\mu_t) = \frac{T-t}{T}\Psi(\mu_0)$$

Suspense Optimum: Construction (2 States)



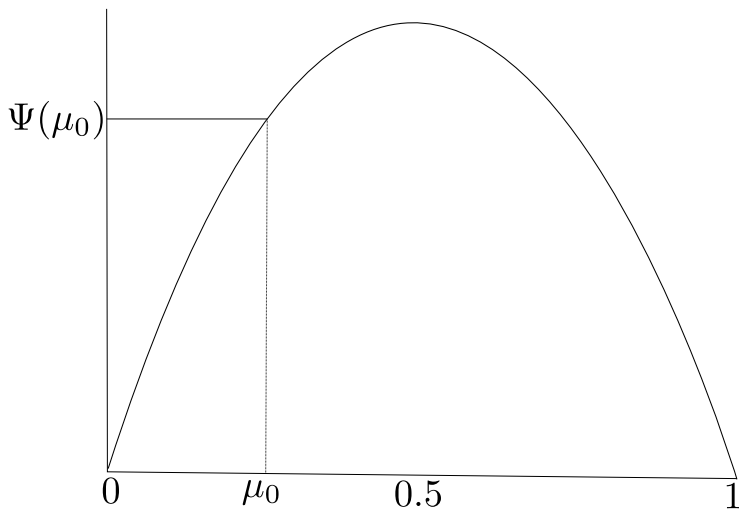
The graph of the function $\Psi(\cdot)$.

Suspense Optimum: Construction (2 States)



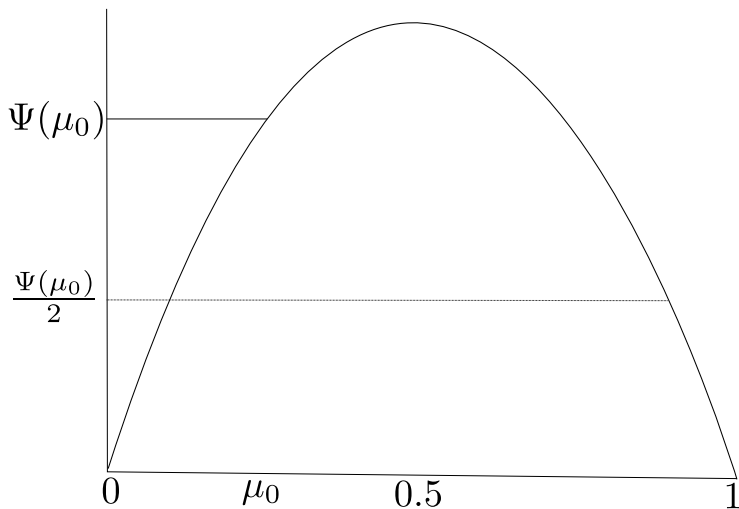
The prior μ_0 .

Suspense Optimum: Construction (2 States)



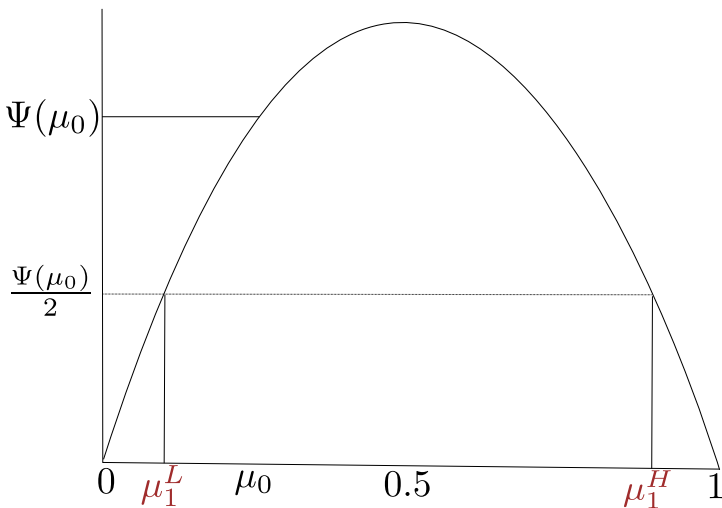
The budget of variance $\Psi(\mu_0)$.

Suspense Optimum: Construction (2 States)



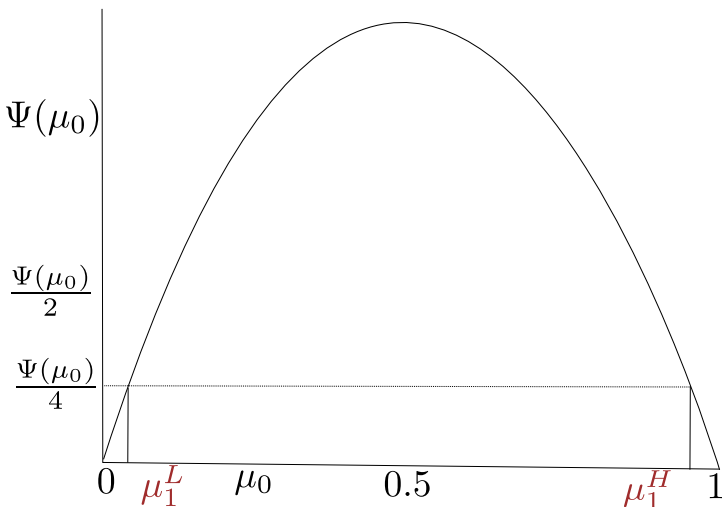
Halfway through the novel, the residual variance must be $\Psi(\mu_0)/2$.

Suspense Optimum: Construction (2 States)



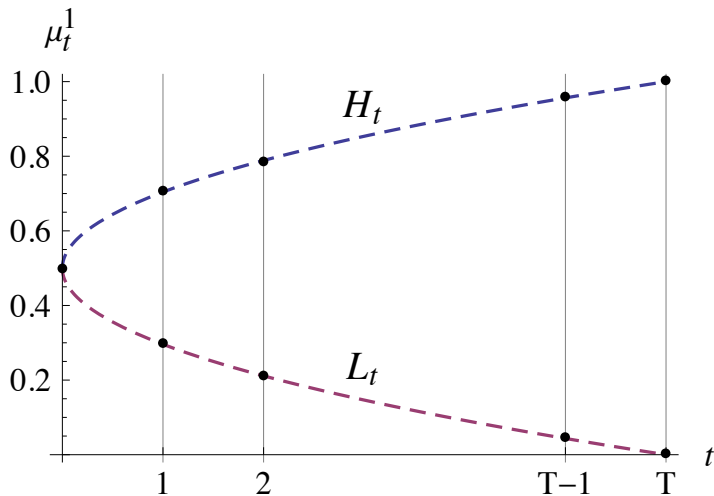
The posterior must therefore be either μ_t^L or μ_t^H .

Suspense Optimum: Construction (2 States)



As the mystery unfolds we either drift closer to the boundary, or experience a *plot twist* that jumps near the other boundary.

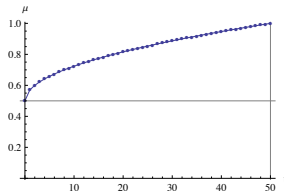
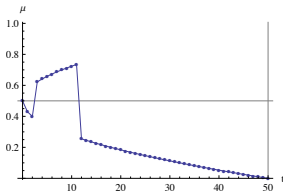
The Suspense-Optimum with Two States



Continuous Time Limit

- Define $s = t/T$, rescaling time.
- Take $T \rightarrow \infty$.
- In the limit we have
 - ▶ Smooth movements toward the boundary
 - ▶ Punctuated by plot twists that arrive according to a Poisson process with arrival rate:

$$\frac{\mu_0(1 - \mu_0)}{1 - 4(1 - s)\mu_0(1 - \mu_0)}.$$



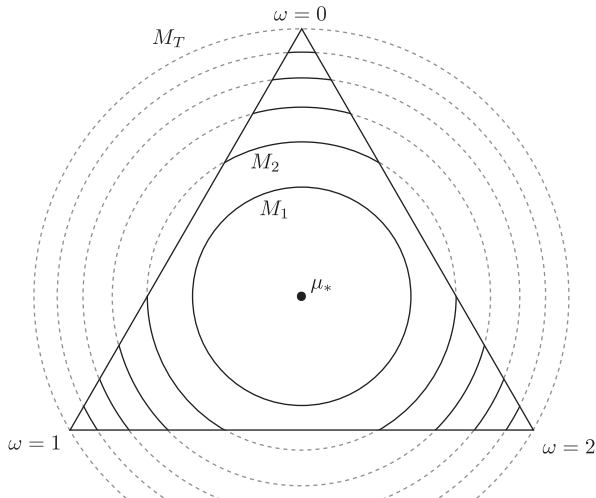
The Suspense-Optimal Sport

In the suspense-optimal sport

- The *last* team to score is the winner.
- It gets harder and harder to score as time goes on.

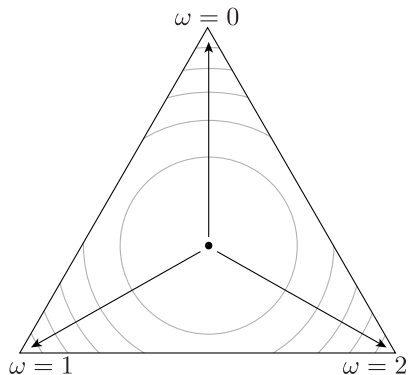
(In sport, *lead changes* are the analog of plot twists.)

3 States

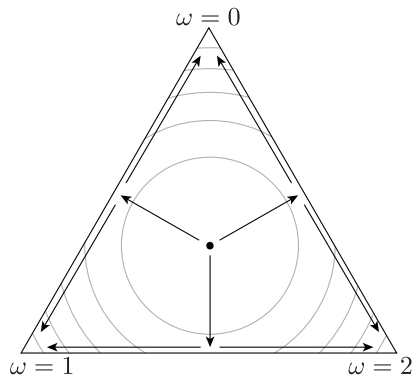


$$M_t = \left\{ \mu_t : \Psi(\mu_t) = \frac{T-t}{T} \Psi(\mu_0) \right\}.$$

Two Examples



(g) Alive Till the End



(h) Sequential Elimination

Qualitative Features

- The state is revealed in the last period, and not before.
- Uncertainty declines over time.
- There is no *ex post* variation in suspense. (no commitment problem.)
- Suspense is constant over time.
- The prior that maximizes suspense is the uniform belief.
- The level of suspense increases in the number of periods T .
- Suspense-optimal information policies are independent of the stage utility function $u(\cdot)$.

Extensions

- Differently weighted states (a protagonist, the home team).
- Principal chooses the weights (Real Madrid and Barcelona)
- Higher weight on later periods.

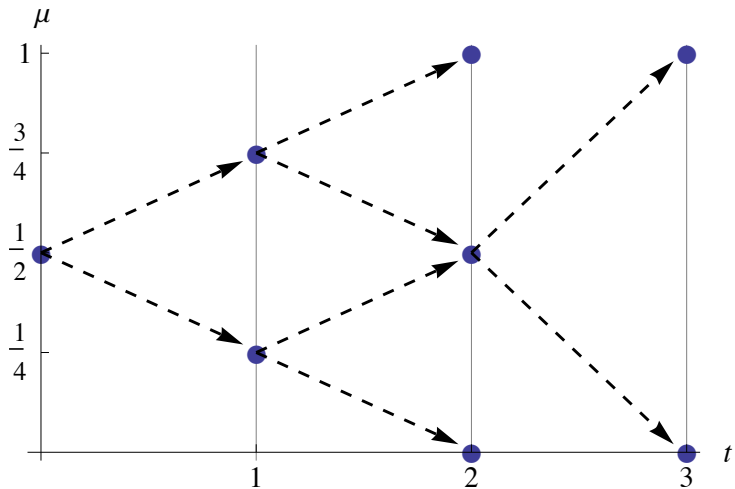
Maximizing Surprise

- Much harder problem
- Focus on $\Omega = \{0, 1\}$ and $u(x) = \sqrt{x}$, i.e.

$$U_{surp}(\eta) = \sum_{t=1}^T |\mu_t - \mu_{t-1}|$$

- Equivalent to maximizing variation of a bounded martingale
 - ▶ Mertens & Zamir (1977); de Meyer (1998)
 - ▶ results in the limit as $T \rightarrow \infty$.
- analytic solution for $T \leq 3$
- numerical solution for larger T
- qualitative features for any T .

Surprise-Optimum for $T = 3$.



Mertens-Zamir (1977)

Proposition

(Mertens & Zamir '77) For any prior $\mu_0 \in [0, 1]$,

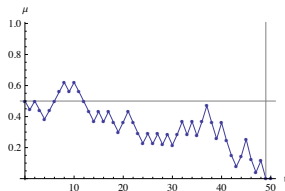
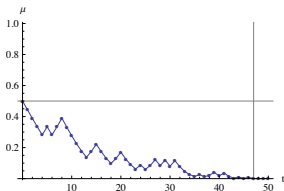
$$\lim_{T \rightarrow \infty} \frac{\max_{\tilde{\mu}} E_{\tilde{\mu}} U_{surp}}{\sqrt{T}} = \phi(\mu_0),$$

where $\phi(\mu_0)$ is the pdf of the standard normal distribution evaluated at its μ_0 -quantile.

A corollary

Proposition

For all $\epsilon > 0$, if $T - t$ is sufficiently large then for any belief path in the support of any surprise-optimal martingale, $|\mu_{t+1} - \mu_t| < \epsilon$.



Qualitative Features

- The state is fully revealed, possibly before the final period.
- Uncertainty may increase or decrease over time.
- Realized surprise is stochastic. (Commitment problem.)
- Surprise varies over time.
- The prior that maximizes surprise is the uniform belief.
- The level of surprise increases in the number of periods T .
- Beliefs change little when there are many periods remaining.
- Belief paths are spiky even when there are many periods.
- Optimum does depend on the utility function $u(\cdot)$.

Constrained Problems

- How should you seed a tournament bracket?
- How many games in a playoff series?
- Who goes first in a political primary, large states or small states?

Future Research

- Better identify preferences for non-instrumental information.
 - ▶ Decision theory
 - ▶ Experimentally
- Measure suspense and surprise of various activities.
 - ▶ Sports (through a structural model of the game)
 - ▶ Gambling (by simulation, eliciting beliefs in the lab.)
 - ▶ Politics (prediction markets)
- Interaction with Decision Making
 - ▶ Gambling
 - ▶ Incentives in sports.

Related Literature

- Optimal release of information
 - ▶ Kamenica Gentzkow (2011)
 - ▶ Horner Skrzypacz (2011)
- Preferences for the timing of the resolution of uncertainty.
 - ▶ Kreps Porteus (1978)
 - ▶ Dillenberger (2010)
 - ▶ Koszegi Rabin (2009)
- Suspense and Surprise
 - ▶ Chan, Courty, Li (2009)
 - ▶ Geanakoplos (1996)
 - ▶ Borwein, Borwein, Marechal (2000)

Fin