Conflict and Cooperation: Bargaining and Commitment

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Motivation

Schelling [4]: When two dynamite trucks meet on a road wide enough for one, who backs up?

These are situations that ultimately involve an element of pure bargaining- bargaining in which each party is guided mainly by his expectations of what the other will accept. But with each guided by expectations and knowing that the other is too, expectations become compounded. A bargain is struck when somebody makes a final, sufficient concession. Why does he concede? Because he thinks the other will not. "I must concede because he won't. He won't because he thinks I will. He thinks I will because he thinks I think he thinks so..." There is some range of alternative outcomes in which any point is better for both sides than no agreement at all. To insist on any such point is pure bargaining, since one always would take less rather than reach no agreement at all, and since one always can recede if retreat proves necessary to agreement. Yet if both parties are aware of the limits to this range, any outcome is a point from which at least one party would have been willing to retreat and the other knows it! There is no resting place.

Bargaining Power: Power to Bind Oneself

- *One-sided commitment*: When one wishes to persuade someone that he would not pay more than $16,000 for a house that is really worth $20,000 to him, what can he do to take advantage of the usually superior credibility of the truth over a false assertion? Answer: make it true.....But suppose the buyer could make an irrevocable and enforceable bet with some third party, duly recorded and certified, according to which he would pay for the house no more than $16,000, or forfeit $5,000.
- [T]he "objective" situation-the buyer's true incentive-has been voluntarily, conspicuously, and irreversibly changed. The seller can take it or leave it. This example demonstrates that if the buyer can accept an irrevocable commitment, in a way that is unambiguously visible to the seller, he can squeeze the range of indeterminacy down to the point most favorable to him

Bargaining Power: Power to Bind Oneself

- *Two-sided commitment*: Interpose some communication difficulty. They must bargain by letter; the invocation becomes effective when signed but cannot be known to the other until its arrival. Now when one party writes such a letter the other may already have signed his own, or may yet do so before the letter of the first arrives. There is then no sale; both are bound to incompatible positions. Each must now recognize this possibility of stalemate, and take into account the likelihood that the other already has, or will have, signed his own commitment.

Myerson's Alternating-Offer Bargaining Model

- Seminal references are Stahl [5] and Rubinstein [3] who offer analyses of alternative-offer models based on time preferences and discounting.
- But, following Schelling's intuitions, a model based on commitment is more natural and compelling. Myerson [2] Section 8.7 offers an alternating-offer bargaining model based on commitment. We offer a simplified treatment of his model.
- There is a pie of size 1. Two players 1 and 2. An *offer* $x = (x_i, x_j)$ with $x_i \in [0,1]$, $x_j = 1 - x_i$ is a division of the pie. Player 1 makes offers in odd periods and player 2 in even periods. In each round, after one player makes an offer, the other accepts or rejects. Acceptance results in offer being implemented and game is over. If player j rejects player i's offer, with probability $p_i \in (0,1)$ the game ends in disagreement and both players get zero.

- Probability p_i is a measure of player i's *commitment power* because if his offer is rejected, the game ends with probability p_i. If $p_i = 1$, the bargaining game is an ultimatum game where player i makes a take-it-or-leave it offer to player j.

Theorem

The Myerson alternating-offer bargaining game has a unique subgame perfect equilibrium in which player i always offers

$$x^i = \left(\frac{p_i}{p_1 + p_2 - p_1 p_2}, 1 - \frac{p_i}{p_1 + p_2 - p_1 p_2}\right)$$

and he accepts an offer that gives him at least x_i^j.

Proof The game is stationary and hence so is the set of subgame perfect equilibrium payoffs. Let M_i be the supremum of player i's payoffs *when he makes an offer;* let m_i be the infimum of player i's payoffs *when he makes an offer.*

Player 2 can expect at most

$$(1 - p_1)\, M_2$$

if he rejects an offer. Hence

$$m_1 = 1 - (1 - p_1)\, M_2 \tag{1}$$

and via a similar argument

$$m_2 = 1 - (1 - p_2)\, M_1. \tag{2}$$

Player 2 can expect at least

$$(1 - p_1)\, m_2$$

if he rejects an offer. Hence,

$$M_1 = 1 - (1 - p_1)\, m_2 \tag{3}$$

and via a similar argument

$$M_2 = 1 - (1 - p_2)\, m_1 . \tag{4}$$

Substituting (4) in (1), we get

$$m_1 = \frac{p_1}{1 - (1 - p_1)\,(1 - p_2)} = \frac{p_1}{p_1 + p_2 - p_1 p_2}.$$

Substituting (2) in (3), we get

$$M_1 = \frac{p_1}{p_1 + p_2 - p_1 p_2}.$$

Hence, $M_1 = m_1 = x_1^1$. Similarly

$$m_2 = M_2 = x_2^2 = \frac{p_2}{p_1 + p_2 - p_1 p_2}.$$

We conclude that in every subgame perfect equilibrium, when player i makes an offer the payoffs must be given by x^i. Player j will accept any offer that gives him more than $(1-p_i)x_j^j = x_j^i$ but this will make player i worse off. Player j will reject anything that gives him less than x_j^i. Player i will then receive $(1-p_j)x_i^j$ which is less than x_i^i. Hence, the subgame perfect equilibrium must have the form in the Theorem. This completes the proof.

Bargaining with Commitment: One-Sided Incomplete Information

- Based on Section 8.8 of Myerson [2].
- With probability $q > 0$, player 1 is a *commitment type* who follows an *r-insistent strategy:* he demands $r \in (0, 1)$ for himself when he makes an offer and accepts offers $y = (y_1, y_2)$ iff $y_1 \geq r$.

Theorem

In any equilibrium, there is a number $J(r, q)$ such that if player 1 plays the r-insistent strategy, player 2 accepts in $J(r, q)$ rounds. Hence, player 1's payoff is at least $r(1 - \max\{p_1, p_2\})^{J(r,q)}$. This is approximately r when p_1 and p_2 are small

Note $J(r, q)$ is not a function of p_1 and p_2.

Proof: Define $\delta = (1 - p_1)(1 - p_2)$. As $1 - r \in (0, 1)$ and $p_1, p_2, q > 0$, we can find integers K and I such that

$$\begin{aligned} \delta^K &< 0.5(1 - r) \\ (1 - 0.5(1 - r))^I &< q. \end{aligned}$$

The proof has two steps.

Step 1: We show that if player 1 plays the $r-$insistent strategy for long enough, player 2 concedes and accepts his demand after a finite number of rounds.

Suppose player 1 has played the $r-$insistent strategy till the present round. Suppose player 2 plans to resist the strategy for the next $2K$ rounds. Let π denote the probability that player 1 will not deviate from the $r-$ insistent strategy in the next $2K$ rounds. Player 2's payoff is at most

$$(1 - \pi(1 - \delta^K)),$$

as bargaining will end with disagreement with at least probability $\pi(1 - \delta^K)$ in the next $2K$ rounds.

Thus for player 2 to resist, we must have

$$1 - r \leq (1 - \pi(1 - \delta^K)) \leq 1 - \pi(1 - 0.5\,(1 - r))$$

so

$$\begin{aligned} \pi &\leq \frac{r}{(1 - 0.5\,(1 - r))} = \frac{1 - 0.5(1 - r) - 0.5(1 - r)}{(1 - 0.5\,(1 - r))} \\ &= 1 - \frac{0.5(1 - r)}{(1 - 0.5\,(1 - r))} \leq 1 - 0.5(1 - r). \end{aligned}$$

By repeating this argument I times, player 2 can resist the $r-$ insistent strategy for $2KI$ rounds if π is less than $(1 - 0.5(1 - r))^I$. But this is less than q and contradicts the fact that $\pi \geq q$. Hence, there is some finite integer L such that player 2 cannot resist the $r-$ insistent strategy for more that L rounds. This completes Step 1.

Step 2: Note L is a function of p_1 and p_2. We work backwards from L to find $J(r, q)$ independent of p_1 and p_2. Before L, if he has never deviated from the $r-$insistent strategy, player 1 knows he can get r if he waits. This limits how much player 2 can extract. For example, with $2k$ rounds to go till L, player 1 will not accept anything less than

$$r(1-p_1)^k(1-p_2)^k = r\delta^k$$

so player 2 gets at most

$$1 - r\delta^k.$$

Let M be an integer such that $M > 2/1 - r$.

Consider round $2Mk$ from L, and suppose player 1 has never deviated from the $r-$insistent strategy. Suppose player 2 plans not to accept the $r-$insistent strategy till L. Let ξ be the probability that player 1 will *not* deviate from the $r-$insistent strategy in the next $2(M-1)k$ rounds. Then, player 2's payoff is at most

$$(1-\xi)\left(1-r\delta^{Mk}\right)+\xi\delta^{(M-1)k}(1-r\delta^{k}).$$

For player 2 to resist till L, we must have

$$1-r\leq(1-\xi)\left(1-r\delta^{Mk}\right)+\xi\delta^{(M-1)k}(1-r\delta^{k})$$

so

$$\xi\leq\frac{r\left(1-\delta^{Mk}\right)}{\left(1-\delta^{(M-1)k}\right)}.$$

By L'Hôpital's Rule and as $M > 2/1 - r$, for all $\delta \in (0, 1)$,

$$\xi \leq \frac{r\left(1-\delta^{Mk}\right)}{\left(1-\delta^{(M-1)k}\right)} \leq \frac{rM}{M-1} \leq 1 - \frac{1}{M-1}.$$

Applying this argument H times with $k = 1, M, M^2, .., M^{H-1}$, we see that the probability that player 1 plays the $r-$insistent strategy cannot be greater than

$$\left(1-\frac{1}{M-1}\right)^H.$$

Suppose H is so large that

$$\left(1-\frac{1}{M-1}\right)^H < q$$

and set $J(r, q) = 2M^H$. Note M and H and hence J are not a function of p_1 and p_2. **End of proof**

- As r is allowed to vary, we can pick out any lower bound on player 1's payoff. And this bound does not depend on p_1 or p_2. Suggests that small changes in bargaining game have large impacts on equilibrium set.
- When p_1 and p_2, barganing is close to efficient.

War of Attrition: Two-Sided Incomplete Information

- What if *both* players have commitment types? Who will concede first when demands are incompatible? The game becomes a *war of attrition.* This has been studied by Abreu and Gul [1].
- Let r^i be commitment type of player i and suppose $r^1 + r^2 > 1$. Let q^i be the probability of type r^i. Player i's rate of time preference is s^i. At time 0, player 1 makes demand r^1, Player 2 either accepts or makes a demand r^2 such that $r^1 + r^2 > 1$. Player 1 can concede or a war of attrition ensues. (This war of attrition can be generated by discrete time games with frequent offers like Myerson game above).
- A strategy for player i is a cumulative distribution $F^i(t)$ with $t \geq 0$ which is his probability of conceding by time t. $F^i(0)$ is the probability that i concedes immediately

Abreu and Gul show there is a unique equilibrium and it has the following three properties:
(i) At most one player concedes with positive probability at time 0;
(ii) After time 0, player i concedes at a constant hazard rate

$$\lambda^i = \frac{s^j \left(1 - r^i\right)}{r^j - \left(1 - r^i\right)}$$

that makes player j indifferent between conceding and waiting and
(iii) There is a finite time T^0 at which the probability that players are irrational simultaneously reaches 1 and concessions stop.
Equilibrium is characterized by the following conditions:

$$\begin{aligned} F^i(t) &= 1 - c^i e^{-\lambda^i t} \text{ for all } t \leq T^0 \\ c^i &\in [0,1], (1-c^1)(1-c^2) = 0 \text{ and} \\ 1 - q^i &= F^i(T^0) \text{ for } i = 1,2. \end{aligned}$$

Note that $1 - c^i = F^i(0)$ so $(1-c^1)(1-c^2) = 0$ is (i). Note the hazard rate $\frac{dF^i/dt}{1-F^i} = \lambda^i$ so this is (ii).

If neither player concedes at time 0 so $c^1 = c^2 = 1$, then the time at which the probability they are irrational reaches one is

$$T^i = -\frac{\log q^i}{\lambda^i}.$$

Let $T^0 = \min\{T^1, T^2\}$. And the final equation is (iii).

Theorem

The unique sequential equilibrium is (F^1, F^2)

Proof Let $(\tilde{F}^1.\tilde{F}^2)$ be a sequential equilibrium. We will argue that it must have the form specified (uniqueness) and that it is an equilibrium (existence).

Uniqueness:

Step 1: The probability that players are irrational reaches 1 simultaneously.

Suppose the probability that player 1 is irrational reaches 1 at time τ^1 before player 2. Then, a rational player 2 is meant to be conceding after τ^1. But this simply delays receiving $1 - r^1$ and cannot be optimal.

Let τ^* be the time where probability that players are irrational reaches 1.

Step 2: Only one player concedes with positive probability at any time (including zero).

If F^1 has a jump at t, then player 2 receives strictly higher utility by conceding the instant after t that by conceding exactly at t.

Step 3: Strategies are strictly *increasing and continuous.*
Suppose player i does not concede over some interval $[t', t'']$ where t'' is the supremum over times where player i's strategy is flat. Then, player j does not concede over this interval as it is better to concede at or before t'. But then player i does strictly better by conceding at t' than just after t'', so $\tilde{F}^i$ is flat at $t'' + \varepsilon$ for $\varepsilon > 0$ and small. This contradicts the definition of t''.
If there is an "atom" at $t > 0$ for player i, player j would not drop out over some interval $(t - \varepsilon, t)$ as he can do better by waiting till the instant after t. But then, player i should drop out at $t - \varepsilon$ not t as this does not reduce the probability of winning and reduces costs of delay. A contradiction.

Uniqueness continued:

Step 4: Indifference....

Players must be indifferent between conceding and waiting at all times $t \in (0, \tau^*]$. Payoff from conceding at t is

$$\int_{x=0}^{t} r^i e^{-s^i x} d\tilde{F}^j(x) + (1 - r^j) e^{-s^i t}(1 - \tilde{F}^j(t)).$$

Differentiating we get

$$0 = r^i e^{-s^i t} \tilde{f}^j(t) - s^i (1 - r^j) e^{-s^i t}(1 - \tilde{F}^j(t)) - (1 - r^j) e^{-s^i t} \tilde{f}^j(t).$$

Hence, $\tilde{F}^j(t) = 1 - c^j e^{-\lambda^j t}$ where c^j is yet to be determined. At τ^*, optimality implies $\tilde{F}^i(\tau^*) = 1 - q^i$. At $t = 0$, if $\tilde{F}^j(0) > 0$, $\tilde{F}^i(0) = 0$ by Step 2. Let T^i solve $1 - e^{-\lambda^i t} = 1 - q^i$. Then, $\tau^* = T^0 = \min\{T^1, T^2\}$ and c^i, c^j are determined by the requirement that $1 - c^j e^{-\lambda^j T^0} = 1 - q^i$. So, $\tilde{F}^i = F^i$.

Existence:

Is player j's strategy is F^j, player i is indifferent between waiting and conceding at all times in the interval $(0, T^0]$ and worse off conceding after T^0. Hence, he is willing to mix according to F^i and (F^i, F^2) is an equilibrium. **End of Proof!!**

Remarks

- Suppose $r^1 = r^2 = r$. Each rational player's expected payoff is $1 - r$. Hence, surplus lost due to delay is

$$1 - 2(1 - r) = 2r - 1 > 0.$$

  In one-sided incomplete information model, there is efficiency: player 1 gets r and player 2 gets $1 - r$.
- Player $i's$ payoff is

$$F^j(0)r^i + (1 - F^j(0)\left(1 - r^j\right).$$

  As $r^i > 1 - r^j$, a player prefers to be conceded to than concede. Player with lower T^i is in stronger position.
- In Myerson model with one-sided incomplete information *and discounting*, as time between offers becomes frequent, delay vanishes and there is efficiency via Coase conjecture logic (Abreu-Gul Lemma 1).

[1] Dilip Abreu and Faruk Gul (2000): “Bargaining and Reputation,” *Econometrica*

[2] Roger Myerson (1991): *Game Theory: Analysis of Conflict*, Harvard University Press, Cambridge MA.

[3] Ariel Rubinstein (1982). "Perfect Equilibrium in a Bargaining Model," *Econometrica* 50 (1): 97–109

[4] Thomas Schelling (1956): “An Essay on Bargaining,” *American Economic Review.*

[5] I. Stahl (1972): *Bargaining Theory*, Stockholm: Stockholm School of Economics.