Conflict and Cooperation: Fear

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Motivation: Schelling and Reciprocal Fear of Surprise Attack

- Thucydides Part 1, Chapter 1 at end: "The growth of Athenian power and the *fear* this caused in Sparta, made war inevitable."
- "[I]f I go downstairs to investigate a noise at night, with a gun in my hand, and find myself face to face with a burglar who has a gun in his hand, there is a danger of an outcome that neither of us desires. Even if he prefers to leave quietly, and I wish him to, there is a danger that he may think I want to shoot, and shoot first. *Worse, there is danger that he may think that I think he wants to shoot. Or he may think that I think he thinks I want to shoot. And so on*" (Schelling, 1960, p. 207).

- A state which does not desire an arms race may in any case acquire new weapons if it fears another state will acquire them:

  "Pakistan does not intend to aggress...[W]e are the victim of (Indian) aggressions." Foreign Minister Gohar Ayub Khan as reported by the Pakistan News Service, June 1999.

  "In India, one often hears that 'Pakistan understands' that India has no hostile designs on it..In Pakistan, however, there is strong sense that the nation's survival is potentially at risk in the event of a major Indian attack. Without a clearer understanding of India's defence doctrine, this could generate a catastrophic miscalculation," CSIS South Asia Monitor, February 1, 1999

  "Whatever happens in India, they blame Pakistan. Whatever happens in Pakistan, we blame India...[N]either Pakistan nor India has gained anything from the conflicts and tensions of the past 25 years." Nawaz Sharif, then

Motivation: Jervis

- The Stag Hunt game is a key metaphor for conflict. Payoffs for player i the row player are:

$$\begin{array}{lcc} & \mathit{Hare} & \mathit{Stag} \\ \mathit{Hare} & h_i - c & h_i \\ \mathit{Stag} & -d & 0 \end{array} \tag{1}$$

with

$$0 > h_i > h_i - c > -d.$$

Note that this requires $c < d$.

- Other key games are Prisoner's Dilemma and Chicken:

$$h_i > 0 > -d > h_i - c.$$

Note this requires $d < c$.

Motivation: Jervis and Stag Hunt

Jervis: This kind of rank-ordering is not entirely an analyst's invention, as is shown by the following section of a British army memo of I903 dealing with British and Russian railroad construction near the Persia-Afghanistan border: The conditions of the problem may . . . . be briefly summarized as follows:
a) If we make a railway to Seistan while Russia remains inactive, we gain a considerable defensive advantage at considerable financial cost;
b) If Russia makes a railway to Seistan, while we remain inactive, she gains a considerable offensive advantage at considerable financial cost;
c) If both we and Russia make railways to Seistan, the defensive and offensive advantages may be held to neutralize each other; in other words, we shall have spent a good deal of money and be no better off than we are at present. On the other hand, we shall be no worse off, whereas under alternative (b) we shall be much worse off. Consequently, the theoretical balance of advantage lies with the proposed railway extension from Quetta to Seistan. W. G.

Motivation: Jervis and Prisoner's Dilemma

Every state is absolutely sovereign in its internal affairs. But this implies that every state must do nothing to interfere in the internal affairs of any other. However, any false or pernicious step taken by any state in its internal affairs may disturb the repose of another state, and this consequent disturbance of another state's repose constitutes an interference in that state's internal affairs. Therefore, every state-or rather, every sovereign of a great power-has the duty, in the name of the sacred right of independence of every state, to supervise the governments of smaller states and to prevent them from taking false and pernicious steps in their internal affairs. Paul Schroeder, *Metternich's Diplomacy at Its Zenith, 182-1823* (Westport, Conn.: Greenwood Press I969), I26.

Motivation: Prisoner's Dilemma or Stag Hunt?

Britain's geographic isolation and political stability allowed her to take a fairly relaxed view of disturbances on the Continent. Minor wars and small changes in territory or in the distribution of power did not affect her vital interests. An adversary who was out to overthrow the system could be stopped after he had made his intentions clear. And revolutions within other states were no menace, since they would not set off unrest within England. Austria, surrounded by strong powers, was not so fortunate; her policy had to be more closely attuned to all conflicts. By the time an aggressor-state had clearly shown its colors, Austria would be gravely threatened. And foreign revolutions, be they democratic or nationalistic, would encourage groups in Austria to upset the existing order. So it is not surprising that Metternich propounded the doctrine summarized earlier, which de- fended Austria's right to interfere in the internal affairs of others, and that British leaders rejected this view.

Motivation: Chicken

The state may be deserted by allies or attacked by neutrals. Or the postwar alignment may rob it of the fruits of victory, as happened to Japan in 1895. Second, the domestic costs of wars must be weighed. Even strong states can be undermined by dissatisfaction with the way the war is run and by the necessary mobilization of men and ideas. Memories of such disruptions were one of the main reasons for the era of relative peace that followed the Napoleonic Wars. Liberal statesmen feared that large armies would lead to despotism; conservative leaders feared that wars would lead to revolution. (The other side of this coin is that when there are domestic consequences of foreign conflict that are positively valued, the net cost of conflict is lowered and cooperation becomes more difficult.) Third- turning to the advantages of cooperation-for states with large and diverse economies the gains from economic exchange are rarely if ever sufficient to prevent war. Norman Angell was wrong about World War I being impossible because of economic ties among the powers; and before World War II, the U.S. was Japan's most important trading partner.

Motivation: Fear and the Security Dilemma

When Germany started building a powerful navy before World War I, Britain objected that it could only be an offensive weapon aimed at her. As Sir Edward Grey, the Foreign Secretary, put it to King Edward VII: "If the German Fleet ever becomes superior to ours, the German Army can conquer this country. There is no corresponding risk of this kind to Germany; for however superior our Fleet was, no naval victory could bring us any nearer to Berlin." The English position was half correct: Germany's navy was an anti-British instrument. But the British often overlooked what the Germans knew full well: "in every quarrel with England, German colonies and trade were . . . hostages for England to take." Thus, whether she intended it or not, the British Navy constituted an important instrument of coercion.

Summary

- In international relations, Stag Hunt, Chicken and Prisoners' Dilemma represent canonical strategic interactions.
- Stag Hunt: aggression feeds on itself and escalates conflict (actions are *strategic complements*).
- Chicken: toughness deters aggression (actions are *strategic substitutes)*
- "World War I was an unwanted spiral of hostility"…"World War II was not an unwanted spiral of hostility-it was a failure to deter Hitler's planned aggression."(Joseph Nye (2007)).
- Stag hunt and chicken have multiple Nash equilibria. Jervis (*spiral model)*, Schelling (*reciprocal fear of surprise attack*): mutual fear and uncertainty determine the outcome.

Private Information

- Independence: Leaders preferences for conflict are independent and unrelated.
- Correlation: Conflict concerns ownership of some resource and payoffs are correlated (c.f. Milgrom-Weber)
- Enough private information (about preferences or costs) selects a *unique* equilibrium. (Unlike global games, no need to assume highly correlated private information.)

- Payoffs for player i (row-chooser)

$$\begin{array}{ccc} & H & D \\ H & h_i - c & h_i \\ D & -d & 0 \end{array} \tag{2}$$

- Strategic complements (Stag Hunt): $(0 <)\ c < d$. Strategic substitutes (Chicken): $c > d\ (> 0)$. We allow h_i to be negative or positive (more soon).
- Player i's *hostility type* h_i is made up of a known publicly known component k_i and a private idiosyncratic component $\eta_i \in [\underline{\eta}, \bar{\eta}]$. So, $h_i = k_i + \eta_i$. Distribution of η_j conditional on $\eta_i = y$ is $F(\cdot|y)$.

Assumption 1. (i) $F_1(x|y) > 0$ (ii) $F_2(x|y) \leq 0$ (a more hostile player i is more pessimistic about j's hostility).
Notice this allows for both independent and positively correlated types.

Strategic Complements

- *Strategic complements:* H more appealing if the opponent is expected to choose H $(c < d)$
- Player i is a *dominant strategy hawk* if $h_i \geq 0$. Player i is a *dominant strategy dove* if $h_i - c \leq -d$ or $h_i \leq c - d$. In between, we have *coordination types.*

Theorem

Suppose both players have dominant strategy hawks and doves. If $d > c$ and

$$F_1(s|t) + F_2(s|t) < \frac{1}{d-c} \tag{3}$$

then the game has a unique BNE. This BNE is in cut-off strategies.

- Independence: "Large idiosyncratic uncertainty" implies uniqueness. This is our formalization of Schelling's "reciprocal fear of surprise attack". Positive correlation helps uniqueness here.

- If player j uses a "cutoff strategy" with cutoff hostility type $h_j = x$, then for player i with hostility type $h_i = y$, the probability player j plays D is $\delta_j(y) = F(x - k_j | y - k_i)$
- Player i's net gain from choosing H instead of D when his type is $h_i = y$ is

$$\Psi^i(x, y) \equiv y + (d - c)\left(1 - F(x - k_j | y - k_i)\right). \qquad (4)$$

- For a cutoff strategy to be a best response, player i should be more inclined to choose H the more hostile he is:

$$\Psi_2^i(x, y) = 1 - (d - c) F_2(x - k_j | y - k_i) > 0 \qquad (5)$$

  In view of Assumption 1, (5) holds if $d > c$. (It also holds if $d < c$ and types are not too strongly correlated.)
- If condition (5) holds then player i's best response to player j's cutoff x is to use a cutoff point denoted $\beta_i(x)$.

- The slope of the best response function is obtained by totally differentiating $\Psi^i(x, \beta_i(x)) = 0$,

$$\beta_i'(x) = -\frac{\Psi_1^i(x, \beta_i(x))}{\Psi_2^i(x, \beta_i(x))} = -\frac{(c-d)F_1(x-k_j|\beta_i(x)-k_i)}{1-(d-c)F_2(x-k_j|\beta_i(x)-k_i)}. \quad (6)$$

Notice that $\beta_i'(x) > 0$ if $d > c$ (and $\beta_i'(x) < 0$ if $d < c$).

- A well-known condition for uniqueness is that the slope is *less than one*. Mathematically, this condition turns out to be $F_1(s|t) + F_2(s|t) < \frac{1}{d-c}$:

$$\begin{aligned} \beta_i'(x) &= \frac{(d-c)F_1}{1-(d-c)F_2} = 1 - \frac{1-(d-c)(F_1+F_2)}{1-(d-c)F_2} \\ &= 1 - \frac{1-(d-c)(F_1+F_2)}{\Psi_2^i(x, \beta(x))} \end{aligned}$$

- We separately show that there are no non-cut-off equilibria

- When there is a cut-off equilibrium, the reaction functions are upward sloping and the incomplete information game is a supermodular game. Hence, when there is a unique equilibrium, it can be obtained by iterated deletion of (interim) dominated strategies (Milgrom and Roberts).
- If (3) holds, then the unique equilibrium is easy to characterize. First, if $\underline{h}_i \geq c - d$ for $i \in \{1, 2\}$ (and $\bar{h}_i > 0$) then there are no dominant-strategy doves, and it is certainly an equilibrium for each player to choose H regardless of type. Thus, this is the *unique* BNE. In a sense, this case represents an extreme case of the "Schelling dilemma". Conversely, if $\bar{h}_i \leq 0$ for $i \in \{1, 2\}$ (and $\underline{h}_i < c - d$) then there are no dominant-strategy hawks, so the unique equilibrium is for each player to choose D, regardless of type.

- If there are both dominant strategy hawks and dominant strategy doves, then certainly neither D nor H can be chosen by all types. Hence, the unique equilibrium must be interior: each player i chooses a cut-off point $h_i^* \in (\underline{h}_i, \bar{h}_i)$ which solves

$$h_i^* + (d - c)\left(1 - F(h_j^* - k_j | h_i^* - k_i)\right) = 0. \qquad (7)$$

- If in addition the players are symmetric, say $k_1 = k_2 = k$, then the best response curves must intersect at the 45 degree line, and the unique equilibrium is a symmetric cut-off equilibrium, $h_1^* = h_2^* = h^*$. The symmetric cut off point is the unique solution in $[\underline{h}_i, \bar{h}_i]$ to the equation

$$h^* + (d - c)\left(1 - F(h^* - k | h^* - k)\right) = 0$$

The comparative statics are as expected. For example, an increase in $d - c$ will lead to more aggressive behavior (a reduction of h^*).

- An interesting special case occurs when types are independent, so $F_2(s|t) = 0$ for all $s, t \in [\underline{\eta}, \bar{\eta}]$. In this case, (3) requires that the density of the random variable η_i is sufficiently spread out, i.e., that there is "enough uncertainty" about types. For example, suppose $k_1 = k_2 = 0$ and η_1 and η_2 are independently drawn from a uniform distribution on $[\underline{\eta}, \bar{\eta}]$. In this case, $F(s|t) = \frac{s-\underline{\eta}}{\bar{\eta}-\underline{\eta}}$, so $F_1(s|t) + F_2(s|t) = \frac{1}{\bar{\eta}-\underline{\eta}} < \frac{1}{d-c}$ if and only if $\bar{\eta} - \underline{\eta} > d - c$. *Thus, with a uniform distribution, there is a unique equilibrium if the support is not too small i.e. if there is enough uncertainty about the opponent's type.*

Strategic Substitutes

- *Strategic substitutes:* H less appealing if the opponent is expected to choose H ($c > d$)
- Player i is a *dominant strategy dove* if $h_i \leq 0$. Player i is a *dominant strategy hawk* if $h_i - c \geq -d$ or $h_i \geq c - d$. Types in between are *opportunistic types*.

Theorem

Suppose both players have dominant strategy hawks and doves. If $c > d$ and for all $x, s, t \in (\underline{\eta}, \bar{\eta})$:

$$F_1(s|t) - F_2(s|t) < \frac{1}{c-d} \quad \text{and} \quad F_1(s|x) - F_2(x|t) < \frac{1}{c-d} \tag{8}$$

then there is a unique BNE. This BNE is a cutoff equilibrium.

- Notice that the assumption $F_1(s|x) - F_2(x|t) < \frac{1}{c-d}$ implies

$$(c-d)F_2(x|t) > (c-d)F_1(s|x) - 1 \geq -1.$$

Therefore, $\Psi_2^i(x, y) > 0$ so a cutoff equilibrium exists.

- Suppose both players have dominant strategy hawks and doves means so that if player j uses cutoff x, $\beta_i(x)$ is interior. Then, we must have $\Psi^i(x, \beta_i(x)) = 0$ and so we can use (6) to study the best-response function β. Since $\Psi_1^i(x, y) > 0$, *the best response functions are downward-sloping:* $\beta_i'(x) < 0$. From (6), we can conclude

$$1 + \beta_i'(x) = 1 + \frac{(d-c)F_1}{1-(d-c)F_2} = \frac{1 + (c-d)(F_2 - F_1)}{1-(d-c)F_2}$$

which is strictly positive as $F_1(s|t) - F_2(s|t) < \frac{1}{c-d}$ by hypothesis. This implies $-1 < \beta_i'(x) < 0$ for all $x \in [\underline{h}_j, \bar{h}_j]$. This implies the two best-response functions cannot cross more than once, so there can be only one cutoff equilibrium.

- The uniqueness conditions for the two classes of games, (8) and (3), can be compared. If types are independent, then $F(s|t)$ is independent of its second argument, so (3) and (8) both reduce to the condition

$$F_1(s|t) < \left|\frac{1}{d-c}\right|$$

for all $s, t \in (\underline{\eta}, \bar{\eta})$. If in addition η_1 and η_2 are independently drawn from a uniform distribution with support $[\underline{\eta}, \bar{\eta}]$, then the sufficient condition for uniqueness of equilibrium is

$$\bar{\eta} - \underline{\eta} > |d - c| .$$

However, if types are affiliated, so that $F_2(s|t) < 0$, then the uniqueness conditions for strategic substitutes are more stringent than those for strategic complements, because $F_2(s|t)$ enters with a *negative* sign in (8).

While stag hunt captured the idea of Schelling's "reciprocal fear of surprise attack," chicken, a game with strategic substitutes captures a different logic of "escalating fear of conflict". Coordination types in chicken want to *mis*-coordinate with the opponent's action, particularly if he plays H. Coordination types with low a low hostility level h are near indifferent between H and D if they are certain that the opponent plays D. But if there is positive probability that the opponent is a dominant strategy type, the "almost dominant strategy doves" strictly prefer to back off and play D. This in turn emboldens coordination types who are almost dominant strategy hawks to play H and the cycle continues. This escalation is more powerful if there is negative correlation between types and dovish coordination types with low h put high probability on hawkish coordination types with high h and vice-versa. But it is more natural to assume independence, if there is no fundamental connection between the two players, or positive correlation, if they are both fighting over a common resource. In the latter case, the uniqueness condition for chicken is less likely to hold.

Global Games: Carlsson and van Damme

- The players' types are generated from an underlying parameter θ as follows. First, θ is drawn from a uniform distribution on $\Theta \equiv [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$. Then, η_1 and η_2 are independently drawn from a uniform distribution on $[\theta - \varepsilon, \theta + \varepsilon]$ (can show conclusions below for "small" noise ε). We assume support of θ is large enough that dominant strategy hawks and doves are feasible. *Neither player can observe* θ.
- If player i draws $\eta_i \in [\underline{\theta} + \varepsilon, \bar{\theta} - \varepsilon]$, then his posterior beliefs about θ are given by a uniform distribution on $[\eta_i - \varepsilon, \eta_i + \varepsilon]$. Therefore, player i's beliefs about η_j are given by a symmetric, triangular distribution around η_i with support $[\eta_i - 2\varepsilon, \eta_i + 2\varepsilon]$.
- You can show

$$F_2(t|s) = F_2(s|t) = -F_1(t|s) = -F_1(s|t).$$

Global Games: Strategic Complements

- For strategic complements, sufficient condition for uniqueness is *always* satisfied:

$$0 = F_1(s|t) + F_2(s|t) < \frac{1}{d-c}.$$

- Also, $F(h|h) = \frac{1}{2}$ for all $h \in [c-d, 0]$ coordination region. CvD show this is obtains approximately for all distribution of θ and all ε as long as ε is small. Hence, in symmetric case

$$\begin{aligned} h^* + (d-c)\left(1 - F(h^* - k|h^* - k)\right) &= h^* + \frac{1}{2}(d-c) \\ &= 0 \end{aligned}$$

so

$$h^* = \frac{1}{2}(c-d).$$

Global Games: Application

- Suppose $\hat{h}$ is the true type of both players and there is complete information. Then, it is possible to sustain (DD) as an equilibrium iff
$$\hat{h} \leq 0.$$
- Now suppose $h_i = \theta + \varepsilon_i$ and that θ is concentrated at $\hat{h}$. Then, (DD) is an equilibrium iff
$$\hat{h} \leq h^* = \frac{1}{2}(c - d) < 0.$$

  It is harder to sustain peace as an equilibrium because of "spiral of fear".

Global Games: Application c'td

- "Predatory" motive h alone matters with complete information. Preemptive motive $c - d$ also matters under incomplete information as this generates "strategic risk". See Chassang and Padro-i-Miquel.
- Suppose h, c and d are function of weapons w held by both players. All three are functions of w. In this symmetric case, (DD) is more likely to be sustainable in the complete information case if (plausibly):

$$h'(w) < 0.$$

- But under incomplete information (DD) is more likely to be sustainable iff

$$h'(w) - \frac{1}{2}(c'(w) - d'(w)) < 0.$$

If the preemption motive increases, it is *harder* to sustain peace.

Global Games: Strategic Substitutes

- For $|s - t| < 2\varepsilon$ we have
$$F_1(s|t) - F_2(s|t) = \frac{1}{2\varepsilon^2}\left(|s - t| + 2\varepsilon\right)$$
which reaches a maximum $2/\varepsilon$ when $|s - t| = 2\varepsilon$.
- Also,
$$F_1(s|x) - F_2(x|t) = F_1(s|x) + F_1(t|x)$$
which also reaches a maximum $2/\varepsilon$. Therefore, our uniqueness condition for strategic substitutes (8) is $\varepsilon > 2\,(c - d)$.

- **Proposition** *If $\varepsilon > 2\,(c - d)$, there is a unique equilibrium when the game has strategic substitutes.*
- However, the global games literature shows that a unique equilibrium exists if the idiosyncratic uncertainty is sufficiently small. We will verify this for our game.

- Assume $k_1 = 0$ and $k_2 = k > 0$ so (D, H) is risk-dominant when both players are opportunistic.
- **Proposition** *If $\varepsilon < k/2$ then there is a unique BNE. In this BNE, player 1 plays H iff $h_1 \geq c - d$ and player 2 plays H iff $h_2 \geq 0$.*

- For intermediate ε, multiple equilibria can exist. Suppose $2\varepsilon > k$. First, there is the above equilibrium where player 2 plays H unless he is a dominant strategy dove and player 1 plays D unless he is a dominant strategy hawk.
- Let $h^* = \frac{(c-d)(2\varepsilon-k)^2}{8\varepsilon^2}$ (and there are some conditions on how high ε can be). Players' strategies are as follows: player 1 plays D iff $h_1 \leq h^*$; player 2 plays D iff $h_2 \leq 0$ or $h_2 \in [h^*, c-d]$.
- Consider player 1 first. For player 1 of type h^*, the probability that player 2 plays H is $F(h^* - k|h^*) = \frac{(2\varepsilon-k)^2}{8\varepsilon^2}$ and he is indifferent between H and D. Higher types are more aggressive and assess a lower probability that player 2 plays H. These types strictly prefer to play H and, by a symmetric argument, lower types prefer to play D.

- We must also show player 2's strategy is at a best-response. Assume $k < 2\varepsilon\left(1 - \frac{2\varepsilon}{c-d}\right)$ .For $h_2 \in \left[h^* + k - 2\varepsilon, h^*\right]$, $\Pr\{h_1 > h^* | h_2\} = \frac{1}{8\varepsilon^2}\left((h_2 - k + 2\varepsilon) - h^*\right)^2$. The net gain from playing H rather than D becomes

$$h_2 + \frac{(d-c)}{8\varepsilon^2}\left(h_2 - h^* - k + 2\varepsilon\right)^2 . \tag{9}$$

  This is quadratic in h_2 and equals zero when $h_2 = h^*$. It reaches a maximum at

$$\hat{h} = h^* + k - 2\varepsilon + \frac{4\varepsilon^2}{c-d}$$

  which is interior to the interval $\left[h^* + k - 2\varepsilon, h^*\right]$ as long as $k < 2\varepsilon\left(1 - \frac{2\varepsilon}{c-d}\right)$. In fact, (9) is strictly positive for $h_2 \in \left[h^* + k - 2\varepsilon, h^*\right)$. For $h_2 \in \left[0, h^* + k - 2\varepsilon\right]$, player 2 knows his opponent plays D and then it is optimal to play H. There is a similar argument for $h_2 \in \left(h^*, c - d\right]$.
- We can reverse roles of players and find a third equilibrium.

[1] Sylvain Chassang and Gerard Padró-i-Miquel: "Conflict and Deterrence under Strategic Risk," forthcoming, *Quarterly Journal of Economics*

[2] Robert Jervis (1978): "Cooperation Under the Security Dilemma", World Politics 30, 167-214.