Conflict and Cooperation: Introduction and Models of Greed

Sandeep Baliga and Tomas Sjöström
Northwestern and Rutgers

Hobbes: Reasons for War

Hobbes [3] Chapter 13, p. 57:

[I]n the nature of man, we find three principal causes of discord. First, *competition (greed)*, secondly *distrust (fear)*, thirdly *glory (honor)*. The first makes men invade for *gain*; the second for *safety*; and the third for *reputation*.

The first use violence to make themselves masters of other men's persons, wives, children, and cattle; the second use it to defend themselves and their families and property·; the third use it for trifles—a word, a smile, a different opinion, and any other sign of a low regard for them personally, if not directly then obliquely through a disrespectful attitude to their family, their friends, their nation, their profession, or their name. This makes it obvious that for as long as men live without a common power to keep them all in awe, they are in the condition known as 'war'; and it is a war of every man against every man.

Thucydides Part 1, Chapter 1 at end: "The growth of Athenian power and the *fear* this caused in Sparta, made war inevitable."

Hobbes: Definitions

Hobbes [3] Chapter 13, p. 57:

Competition: if any two men want a single thing which they can't both enjoy, they become enemies; and each of them on the way to his goal (which is principally his own survival, though sometimes merely his delight) tries to destroy or subdue the other.

Distrust: Because of this distrust amongst men, the most reasonable way for any man to make himself safe is to strike first...People who would otherwise be glad to be at ease within modest bounds have to increase their power by further invasions, because without that, in a purely defensive posture, they wouldn't be able to survive for long.

Glory: Every man wants his associates to value him as highly as he values himself; and any sign that he is disregarded or undervalued naturally leads a man to try, as far as he dares, to raise his value in the eyes of others....when there is no common power to keep them at peace, 'as far as he dares' is far enough to make them destroy each other.

Hobbes: Economic Consequences of "Hold Up"

Hobbes [3] Chapter 13, p. 57-58:
[T]here is no place for hard work, because there is no assurance that it will yield results; and consequently no cultivation of the earth, no navigation or use of materials that can be imported by sea, no construction of large buildings, no machines for moving things that require much force, no knowledge of the face of the earth, no account of time, no practical skills, no literature or scholarship, no society; and—worst of all—continual fear and danger of violent death, and the life of man solitary, poor, *nasty, brutish, and short*.
Empirical evidence:
For the savage people in many parts of America have no government at all except for the government of small families, whose harmony depends on natural lust. Those savages live right now in the brutish manner I have described.

Hobbes: Optimal (Individual) Strategies and Coordination

Hobbes [3] Chapter 14, p. 60: The *First Law of Nature*
[A]s long as every man continues to have this natural right to everything—no man, however strong or clever he may be, can be sure of living out the time that nature ordinarily allows men to live. And consequently it is a command or general rule of reason that *every man ought to seek peace, as far as he has any hope of obtaining it*; and that *when he can't obtain it he may seek and use all helps and advantages of war*.

Hobbes: Optimal (Collective) Strategies: Leviathan

Hobbes [3] Ch. 17, p. 79

The only way to establish a common power that can defend them from the invasion of foreigners and the injuries of one another, and thereby make them secure enough to be able to nourish themselves and live contentedly through their own labours and the fruits of the earth, is to confer all their power and strength on one man, or one assembly of men, so as to turn all their wills by a majority vote into a single will...This is the method of creation of that great LEVIATHAN, or rather (to speak more reverently) of that mortal god to which we owe, under the immortal God, our peace and defence.

Summary

- *State of nature* suggests anarchy is a Prisoners' Dilemma because of *greed*.
- *First Law of Nature* suggests peace is a best response to peace and war to war, a Coordination game. It suggests a channel through which *fear* may operate.
- *Leviathan* offers a theory of government.

Guns vs. Butter

- Based on papers by Brito and Intriligator, Hirshleifer and Skaperdas.
- There are two players (countries), A and B. Player $i \in \{A, B\}$ has has the potential to produce output k_i. The output can be either guns (w_i) (Hirshleifer: "fighting effort") or butter (x_i) ("productive effort"). The *budget constraint* is

  $$k_i = w_i + x_i$$

  There is a non-negativity constraint:

  $$w_i \geq 0 \quad , \quad x_i \geq 0$$

Guns can be used to fight a war. The winner takes all the butter: he consumes $x_A + x_B$. The loser gets no butter. Player i wins the war with probability

$$p(w_i, w_j) \equiv \frac{w_i}{w_i + w_j}.$$

Guns vs. Butter c'td

- The function p is the *contest success function* (CSF). Equivalently, assuming risk-neutrality, country i gets a fraction $p(w_i, w_j)$ of the total wealth $(x_A + x_B)$).
- A more general contest success function is

$$p(w_i, w_j) = \frac{\phi(w_i)}{\phi(w_i) + \phi(w_i)}$$

  where ϕ is an increasing function. For an axiomatization, see Skaperdas, S. (1996) "Contest Success Functions," *Economic Theory* 7: 283-290.
- To make war an inefficient outcome, assume player i suffers a cost $c_i > 0$ from a war.
- If $c_A > k_B$ and $c_B > k_A$: there is an efficient equilibrium with no war.
- To simplify, *assume*

$$c_A \geq k_B \text{ and } c_B < k_A$$

  so country A could never benefit from a war but B could. Think of A as "soft and rich" and B as "tough and poor"

Guns vs. Butter c'td

Time line:

- 1: A and B produce guns and butter (these decisions are publicly observed).
- 2: B decides whether or not to attack A.

Guns vs. Butter c'td

Theorem

There must be an attack by B with positive probability in equilibrium.

Proof.

Using our assumption, there is no equilibrium where neither players arms as B has the incentive to arm if A is unarmed.
There is no equilibrium where B does not attack with positive probability. Otherwise, B does not acquire arms in equilibrium.
But if B does not acquire arms in equilibrium, nor does A. But we already claimed this is not an equilibrium. □

Guns vs. Butter: Preliminary Conclusions

- This model can explain both *arms races* and *war*.
- War is driven by *greed* in the absence of property rights.
- There is also an element of *fear*: B may acquire arms not to attack A but to defend himself.
- There is inefficiency as guns which cannot be "consumed" are produced rather than butter and because war is costly.

Coase Theorem

Coase [1] p. 2 and 8 resp.

- Another example is afforded by the problem of straying cattle which destroy crops on neighbouring land. If it is inevitable that some cattle will stray, an increase in the supply of meat can only be obtained at the expense of a decrease in the supply of crops.
- It is necessary to know whether the damaging business is liable or not for damage caused since without the establishment of this initial delimitation of rights there can be no market transactions to transfer and recombine them. But the ultimate result (which maximizes the value of production) is independent of the legal position if the pricing system is assumed to work without cost.
- Fearon [2] introduces this perspective into political science and also offers theories for why Coase Theorem may fail.

Guns vs. Butter with Transfers

Time line:

- 1: A and B produce guns and butter (these decisions are publicly observed).
- 2: A voluntarily transfers an amount t of butter to B.
- 3: B decides whether or not to attack A.

As transfers are possible, and there is complete information, the equilibrium outcome will be *ex post* efficient: no war in equilibrium. But the equilibrium outcome is *ex ante inefficient* if $w_i > 0$ in equilibrium.

Guns vs. Butter with Transfers: Equilibrium

We solve by backward induction. At time 3, B will not attack if

$$\frac{w_B}{w_A + w_B}(x_A + x_B) - c_B \leq x_B + t \tag{1}$$

There are two possibilities. There is *deterrence* if

$$\frac{w_B k_A - w_A k_B}{w_A + w_B} - c_B \leq 0 \tag{2}$$

In this case (1) holds with $t = 0$ (using the budget restrictions to substitute for x_A and x_B), so no need for A to make any transfer. But if (2) is violated, then to avoid a conflict, A makes the smallest non-negative transfer such that (1) holds, which is

$$t = \frac{w_B k_A - w_A k_B}{w_A + w_B} - c_B > 0 \tag{3}$$

This is called *appeasement.*

In general, A will pay

$$t = \max\left\{0,\ \frac{w_B k_A - w_A k_B}{w_A + w_B} - c_B\right\} \tag{4}$$

Guns vs. Butter with Transfers: Equilibrium c'td

Theorem

Suppose

$$k_B < \frac{1}{3} k_A \tag{5}$$

and

$$c_B < \sqrt{k_B \left(k_A + k_B\right)} - 2k_B \tag{6}$$

There is an equilibrium where A sets

$$w_A = \sqrt{k_B(k_A + k_B)} - k_B < k_A \tag{7}$$

and B sets

$$w_B = k_B. \tag{8}$$

Guns vs. Butter with Transfers: Equilibrium c'td

Proof: We show this is an equilibrium. B in effect has two choices. He can either choose $w_B = 0$, and consume $x_B = k_B$. Or, he can choose $w_B > 0$ in the appeasement region, i.e., such that $t > 0$, where t is given by (4). (To set $w_B > 0$ such that $t = 0$ is evidently useless). With appeasement, B's payoff is

$$k_B - w_B + t = \frac{w_B}{w_A + w_B}(k_A + k_B) - w_B - c_B$$

The derivative of this expression w.r.t. w_B is

$$\frac{w_A}{(w_A + w_B)^2}(k_A + k_B) - 1.$$

Substituting in for w_A from (7), this is positive for all $w_B \leq k_B$ as long as (5) holds. So, the best option for B in the appeasement region is $w_B = k_B$. Moreover, he is better off setting $w_B = k_B$ than $w_B = 0$ as long as (6) holds.

Thus, (8) is B's best option.

Now, consider player A. There is deterrence if (2) holds. Clearly, it is useless to set w_A so high that (2) holds strictly. Thus,

$$w_A \leq \frac{k_B(k_A - c_B)}{c_B + k_B}$$

For any such w_A, (3) and (8) imply that A will pay the transfer

$$t = \frac{k_A - w_A}{w_A + k_B} k_B - c_B \tag{9}$$

and get payoff

$$x_A - t = \frac{w_A}{w_A + k_B}(k_A + k_B) - w_A + c_B \tag{10}$$

The derivative of this expression w.r.t. w_A is zero when (7) holds. Moreover, this is in the appeasement region because

$$t = \sqrt{k_B (k_A + k_B)} - k_B - c_B > 0$$

by (6). Thus, (7) is A's best option.

Exercise: Find equilibria of model with transfers when $c_A < k_B$ and $c_B < k_A$.

Guns vs. Butter with Transfers: Conclusions

- This model can explain arms races, but it cannot explain wars, since there is no war in equilibrium.
- Since $k_B < k_A/3$,

$$w_A = \sqrt{k_B(k_A + k_B)} - k_B > k_B = w_B$$

  Therefore, the rich "soft" country A spends more on its military than the poor "tough" country B. But, A still has to appease B by handing over butter to prevent an attack.

Guns vs. Butter with Transfers: Conclusions

- Suppose world output is normalized, $k_A + k_B \equiv 1$. So $w_A = \sqrt{k_B} - k_B$. Lets lets transfer productive capital from rich A to poor B, i.e., lets increase k_B (while maintaining $k_A + k_B = 1$). Since $k_B < (1 - k_B)/3$ implies $k_B < 1/4$,

$$\frac{\partial w_A}{\partial k_B} = \frac{1}{2\sqrt{k_B}} - 1 > \frac{1}{2\sqrt{1/4}} - 1 = 0$$

  and $\frac{\partial w_B}{\partial k_B} = 1 > 0$. So if we reduce inequality then the arms race intensifies: *both* countries get more guns. The transfer of butter from A to B increases as B gets richer:

$$\frac{\partial t}{\partial k_B} = \frac{\partial}{\partial k_B}\left(\sqrt{k_B} - k_B - c_B\right) = \frac{1}{2\sqrt{k_B}}\left(1 - 2\sqrt{k_B}\right) > 0 \tag{11}$$

  as $k_B < 1/4$. Redistribution of capital from A to B makes A unambiguously worse off. Computation shows that B prefers redistribution, even though the arms race intensifies.

Why Do We Have War?

In economics, we have two standard theories of *transactions costs* that cause the failure of the Coast Theorem:

1. *Nontransferable utility*
2. *Asymmetric Information* - Myerson and Satterthwaite

To this we can add a third:

3. *Non-stationary Game/ "Shifting Power"* - Acemoglu and Robinson, and Fearon.

Asymmetric Information and Signaling

Following Brito and Intriligator, we introduce incomplete information into the model. Suppose B privately knows his own "type", which is his cost of war. B's type is either c_B or $\bar{c}$, where $c_B < k_A < \bar{c}$. B is *tough* if his type is c_B and *weak* if it is $\bar{c}$. A weak B would never attack, but a tough B might. The probability that his type is c_B is p. Let $w_B = w_B(\tau)$ be the amount of guns that B produces if his type is $\tau \in \{c_B, \bar{c}\}$.

Time line:

- 0: B privately observes his own type.
- 1: A and B produce guns and butter (these decisions are publicly observed).
- 2: A voluntarily transfers an amount t of butter to B.
- 3: B decides whether or not to attack A.

Asymmetric Information and Signaling: Equilibrium

Since w_B is observed by A, it is a signaling model. There is no separating equilibrium where $w_B(\bar{c}) \neq w_B(c_B)$. For in this case, when A observes $w_B(c_B)$ he knows B is tough and A makes a transfer to avoid war. But then B's weak type would also choose $w_B(c_B)$ and take the transfer.

Asymmetric Information and Signaling: Equilibrium continued

Theorem *Suppose (5) and (6) hold, and in addition*

$$c_B + pc_A < (1-p)\left(\sqrt{k_B(k_A+k_B)} - k_B\right). \tag{12}$$

There exists a semi-separating equilibrium *where B's tough type sets $w_B = k_B$ for sure. The weak type sets $w_B = k_B$ ("bluffing") with probability q, where*

$$q = \frac{p}{1-p}\frac{c_B + c_A}{\sqrt{k_B(k_A+k_B)} - k_B - c_B}, \tag{13}$$

and $w_B = 0$ with probability $1-q$.

A sets

$$w_A = \sqrt{k_B(k_A + k_B)} - k_B, \qquad (14)$$

and when he sees $w_B = k_B$ *he chooses appeasement with probability* τ, *where*

$$\tau = \frac{k_B}{\sqrt{k_B(k_A + k_B)} - k_B - c_B}. \qquad (15)$$

With probability $1 - \tau$ *A "calls B's bluff" and makes no transfer. In equilibrium, a war occurs with probability*

$$W = p(1 - \tau) = p - \frac{pk_B}{\sqrt{k_B(k_A + k_B)} - c_B - k_B} > 0 \qquad (16)$$

Asymmetric Information and Signaling: Equilibrium c'td

Proof: First, you can verify that under our assumptions (12) and (6), (13) and (15) imply $0 < q < 1$ and $0 < \tau < 1$.
Next, we need to check that each player plays optimally. First, suppose A observes $w_B = k_B$. He thinks B is tough with probability $\frac{p}{p+(1-p)q}$. Since A randomizes in this equilibrium, he must be indifferent between appeasement and no appeasement. If A chooses appeasement, he pays t given by (9) and A's payoff is

$$x_A - t = \frac{w_A}{w_A + k_B}(k_A - w_A) + c_B \tag{17}$$

If there is no appeasement, then either B is tough and A gets $\frac{w_A}{w_A+k_B}(k_A - w_A) - c_A$, or B is weak and A gets $k_A - w_A$, so A's expected payoff is

$$\frac{p}{p+(1-p)q} \times \left[\frac{w_A}{w_A + k_B}(k_A - w_A) - c_A\right] + \frac{(1-p)q}{p+(1-p)q}(k_A - w_A) \tag{18}$$

Asymmetric Information and Signaling: Equilibrium c'td

Using (13) and (14), we find that (17) equals (18). Thus, A is indifferent between appeasement and no appeasement. For his choice of w_A, the argument is the same as in the proof of Proposition **??**.

Next, consider the weak type of B. If he sets $w_B = 0$, his payoff is k_B. If he sets $w_B = k_B$, he receives t with probability τ, and 0 with probability $1 - \tau$. His expected payoff is τt. Now (9), (14) and (15) imply that $\tau t = k_B$. Thus, the weak type of B is indifferent between $w_B = 0$ and $w_B = k_B$.

If the weak type is indifferent between $w_B = 0$ and $w_B = k_B$, the tough type certainly prefers $w_B = k_B$.

Finally, if B chooses anything else than $w_B = 0$ and $w_B = k_B$, we may assume A believes B is weak and therefore refuses to make a transfer. Given this, nothing except $w_B = 0$ or $w_B = k_B$ can be an optimal choice for B. **End of proof**

Exercise: Find *all* the equilibria of this game. Do they all involve war with positive probability?

Asymmetric Information and Signaling: Conclusions

- This model can explain both arms races and wars, since there is war in equilibrium. A doesn't know if B is weak (and unwilling to fight) or tough (and willing to fight). With some probability, the weak type mimics the tough by arming himself ("bluffing"). With positive probability, A calls the bluff, B backs down if he is weak, but if B is truly tough, a war ensues.
- Is equality good for peace? Normalize $k_A + k_B = 1$. The probability of war is determined by τ, the probability that A calls B's bluff, which is determined by the indifference condition for the weak type: $t\tau = k_B$. Both t and k_B increase if we redistribute capital from A to B, and the effect on τ is ambiguous. Formally,

$$\frac{\partial W}{\partial k_B} = \frac{p}{2} \frac{2c_B - \sqrt{k_B}}{\left(k_B + c_B - \sqrt{k_B}\right)^2}$$

which could be positive or negative. If c_B is very small then $\frac{\partial W}{\partial k_B} < 0$ and equality is good for peace.

Coalitions and Greed

- Based on paper of James Jordan [4]
- "Wealth is Power"
- The world consists of a set N of agents. A coalition is a set $C \subseteq N$. Agent i's wealth is w_i. By a normalization, $\sum_{i \in N} w_i = 1$.
- Any coalition with more than half the wealth can take everything by force, at no cost to itself.
- Main Question: What wealth distributions can persist in such a world?

Coalitions and Greed c'td

A wealth distribution $w = (w_1, \ldots, w_n)$ is *dominated* if there is a coalition C and wealth distribution w' such that

$$w'_i > w_i \quad \text{for all } i \in C \tag{19}$$

and

$$\sum_{i \in C} w_i > 1/2 \tag{20}$$

Here (19) means C prefers w' to w and (20) means C is strong enough to get what it wants *even when every player outside C is mobilized against it.*

Coalitions and Greed: Example

Consider a simple world with three agents. Clearly, if one agent i has all the wealth, $w_i = 1$ for some i, then this is undominated. Also, if two agents have 50% and the third zero, $w_i = w_j = 1/2$ and $w_k = 0$, the outcome is also undominated. These are the *only* undominated outcomes. To see this, suppose that no agent has 50% of all the wealth, i.e., $w_i < 1/2$ for all i. Then any two-agent coalition $C = \{j, k\}$ has more than 50% and can take everything by force. The remaining player i must end up with $w_i = 0$. But since j and k are arbitrary, this means $w_i = 0$ for all i, a contradiction. This argument generalizes to more than three players.

Coalitions and Greed: Example c'td

Hence:

Theorem

There are two kinds of undominated wealth distributions: those where one agent owns everything, and those where two agents own 50% each.

Thus, according to this criterion, only extreme concentrations of wealth, in which one player owns everything (Roman Empire?) or two players each own half of everything (East vs. West in Cold War?) can persist.

Coalitions and Greed: Example and vNM Stable Sets

- This criterion is too simplistic. For example, consider the wealth distribution $w = (1/2, 1/4, 1/4)$. It is dominated by $w' = (0.6, 0.4, 0)$, via the coalition $C = \{1, 2\}$. However, agent 2 would be stupid to enter into this coalition with 1. Agent 2 should understand that, once w' is reached, agent 1 will go on to take everything and 2 will end up with 0. We need some criterion of "far-sighted stability".
- This criterion is the *stable set*, the von Neumann-Morgenstern solution. A stable set is an "accepted standard of behavior". Formally a set S of wealth distributions is stable if it satisfies two conditions.
  - (Internal stability) No allocation $w \in S$ is dominated by any other allocation $w' \in S$;
  - (External Stability) Every allocation $w \notin S$ is dominated by some allocation $w' \in S$.

Coalitions and Greed: Theorem

What sets are stable in the three agent world? External stability implies that every undominated allocation must belong to every stable set, so every stable set must include the allocations (1, 0, 0) and (1/2, 1/2, 0) and their permutations. Could the set of all undominated outcomes be externally stable ? No because distributions where $w_i = 1/2$ for some i, and $w_j > 0$ and $w_k > 0$ are not dominated by any undominated allocation. For example, consider $w = (1/2, 1/4, 1/4)$. Suppose it is dominated by w' via some coalition C strong enough to enforce w'. Then it must be the case that $C = \{1, j\}$ or else C cannot have more than half the wealth and cannot enforce w'. Moreover, $w'_1 > w_1 = 1/2$ and $w'_j > 1/4$. But then, by our theorem, w' is not undominated. So lets include all such distributions: that yields a set $\hat{S}$.

Theorem

(Jordan) $\hat{S}$ is the unique stable set.

Coalitions and Greed: Proof of Theorem

It is easy to check that $\hat{S}$ is both internally and externally stable, hence it is a stable set. It remains to show it is the *unique* stable set.

Since every undominated allocation must belong to every stable set, no outcome which is dominated by an undominated allocation can belong to a stable set (that would violate internal stability). Allocations of the form $(1/2, w_2, w_3)$ (with $w_2 > 0$ and $w_3 > 0$) can only be dominated by allocations of the form $(1/2 + \varepsilon, w_2 + \varepsilon, 0)$, up to permutations. But $(1/2 + \varepsilon, w_2 + \varepsilon, 0)$ is dominated by the undominated allocation $(1, 0, 0)$ so $(1/2 + \varepsilon, w_2 + \varepsilon, 0)$ cannot be included in a stable set. Therefore $(1/2, w_2, w_3)$ must be included in every stable set. This proves every stable set must include $\hat{S}$. But $\hat{S}$ is externally stable, so including additional allocations together with $\hat{S}$ would violate internal stability.

Coalitions and Greed: Final Observations

- The stable set remains unique in games with more than three players. The undominated allocations are (1, 0, ... , 0) and (1/2, 1/2, 0, ... , 0) and permutations. The stable set again contains the undominated allocations and the allocations $(1/2, w_2, ..., w_n)$ and permutations. However, there are additional stable allocations of the form (2^{-j} , ... , 2^{-j} , 0 . . . , 0) and permutations, for each $j \leq \log_2 n$.
- For example, with four players, the stable set includes the allocation $w = (1/4, 1/4, 1/4, 1/4)$. Three players, say 1,2 and 3, can form a coalition to pillage a fourth and they all must get strictly more than 1/4. But the resulting allocation is dominated by $(1/2, 1/2, 0, 0)$ and violates external stability.
- That wealth distributions such as (1/4, 1/4, 1/4, 1/4) can be stable "standards of behavior" should be a cause for celebration.

- Jordan [5] investigates the structure of stable sets when not everyone outside the pillaging coalition C is mobilized against it. And he allows "power in numbers" as well as in wealth. As it is "easier" for a coalition to stage an attack, the stable set is smaller. It is still unique.

[1] Ronald Coase (1960): "The Problem of Social Cost", *Journal of Law and Economics*, Vol. 3, pp. 1-44.

[2] James Fearon (1995): "Rationalist Explanations for War", *International Organization*, Vol. 49, No. 3, pp. 379-414.

[3] Thomas Hobbes (1651): *Leviathan, or the Matter, Forme, and Power of a Commonwealth, Ecclesiasticall and Civil,* http://www.earlymoderntexts.com/f_hobbes.html

[4] James Jordan (2001): "Majority Rule with Dollar Voting," *Review of Economic Design*, 6: 343-352.

[5] James Jordan (2006): "Pillage and Property", *Journal of Economic Theory*, Vol. 131, No. 1 , November, pp. 26-44.