Utilitarianism

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Why Utilitarianism?

- We saw last time that any standard of social welfare is problematic in a precise sense.
- If we want to proceed, we need to compromise in some way.
- We will compromise by focusing on environments where intensity of preference can be measured.

Money

- We will assume that *monetary transfers* are possible and can be enforced.
- A monetary transfer scheme can be represented by $t = (t_1, \ldots, t_n)$ where
  - t_i denotes the amount of money paid *to* individual i. (could be negative)
  - $\sum_{i=1}^{n} t_i = t_1 + t_2 + \ldots + t_n$ is the *budget deficit.* (could be negative)
  - $\sum_{i=1}^{n} t_i = 0$ means that the transfer scheme has a *balanced budget.*

Social Choices with Monetary Transfers

- Remember that society must choose an alternative.
- Now alternatives have two components.
  - A choice from $\mathcal{A}$ (e.g. who gets to attend the NIN concert and who doesn't)
  - A monetary transfer scheme t (i.e. who pays, who gets paid, and how much.)
- And now we must describe the individuals' preferences over both components. (i.e. how do they trade-off monetary payments versus better/worse alternatives.)

Money Preferences

Thought experiment.

- Pile of money.
- Ticket to see NIN.

How large can we make the pile of money before you choose that over the NIN ticket?

We equate that with your *willingness to pay.*

Money Utility

Willingness to pay is captured by utility functions that work as follows.

Definition

The *value* to individual i from alternative x is denoted $v_i(x)$. The *utility* associated with alternative x together with monetary transfer t_i is

$$U_i(x, t_i) = v_i(x) + t_i$$

Individual i prefers a pair (x, t_i) to a pair (y, t_i') if $U_i(x, t_i) \geq U_i(y, t_i')$ and if the inequality is strict, we say his preference is *strict.*

As always in economics, a utility function is just a mathematical device that allows us to describe preferences in a precise way.
Let's verify that a utility function like U_i describes wilingness to pay.

Money Utility and WTP

Example

Suppose there is one ticket left to the NIN show. Alternative x is you get it, alternative y is I get it. Suppose that you derive no value from *me* seeing the show, so $v_{\text{you}}(y) = 0$ and that your value from seing the show is $v_{\text{you}}(x)$ (some number.) If you are asked to choose between seeing the show (x) and paying t dollars versus not seeing the show (y) and paying nothing, you would be willing to pay whenever

$$U_{\text{you}}(x, -t) \geq U_{\text{you}}(y, 0)$$

which translates to

$$v_{\text{you}}(x) + (-t) \geq 0$$

or

$$t \leq v_{\text{you}}(x)$$

This says that you are willing to pay (up to but no more than) $v_{\text{you}}(x)$ to see the show.

More on WTP

More generally, if x and y are any two alternatives, and t is a number, individual i prefers $(x, -t)$ to $(y, 0)$ whenever

$$U_i(x, -t) \geq U_i(y, 0)$$

which translates to

$$t \leq v_i(x) - v_i(y)$$

so that $v_i(x) - v_i(y)$ measures i's willingness to pay to have x rather than y. (And this may be negative.)

The Utilitarian Social Welfare Function

Definition

Under the utilitarian social welfare function, society prefers (x, t) to (y, t') if $\sum_{i=1}^{n} U_i(x, t_i) \geq \sum_{i=1}^{n} U_i(y, t_i')$. In particular, if t and t' have a balanced budget then this reduces to

$$\sum_{i=1}^{n} v_i(x) \geq \sum_{i=1}^{n} v_i(y)$$

This social welfare function satisfies IIA and Pareto and is not a dictatorship.

Not Perfect

- Willingness to accept vs. willingness to pay. (and ability to pay.)
- Arguably not comparable across people.
- Time rather than money?

Pareto Efficiency

For now, we restrict attention to monetary transfer schemes that have a balanced budget.

Definition

Social choice (x, t) is *Pareto dominated* by another social choice (y, t') if every individual prefers (x, t) to (y, t') and at least one individual strictly prefers it.

Definition

A social choice (x, t) is *Pareto efficient* if there is no (y, t') that Pareto dominates it.

In the absence of monetary transfers, there will typically be many Pareto efficient alternatives.

Utilitarianism and Pareto Efficiency

When monetary transfers are possible, there will typically be just one Pareto efficient alternative.

Definition

If x is an alterantive for which $\sum_i v_i(x) \geq \sum_i v_i(y)$ for every alternative y, then x is called the Utilitarian alternative.

Proposition

When monetary transfers are possible, if (x, t) is Pareto efficient, then x must be utilitarian as well.

To demonstrate this we will show that if x is not utilitarian, then (x, t) is not Pareto efficient.

Utilitarianism and Pareto efficiency

Suppose that x is not utilitarian, i.e. there is some y such that

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Then it is possible to devise a monetary transfer scheme $\tilde{t}$ so that $(y, \tilde{t})$ Pareto dominates (x, t). To do so, first define

$$\hat{t}_i = v_i(x) - v_i(y)$$

(Note that this is positive for those who like x better than y, negative otherwise.)

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And now, set $\tilde{t}_i = t_i + \hat{t}_i$. We see that everyone is indifferent.

$$U_i\left(y, \tilde{t}_i\right) = v_i\left(y\right) + \tilde{t}_i$$

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Utilitarianism and Pareto efficiency

But notice that $\tilde{t}$ has a budget surplus:

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which is negative by our original assumption. Thus it is possible to add a small amount to every individual's transfer so that y (and the transfer) will Pareto dominate (x, t).

Utilitarianism and Pareto efficiency

Proposition

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Consider any alternative y and balanced-budget transfer scheme $\hat{t}$. Suppose $(y, \hat{t})$ were Pareto superior to (x, t). That means,

$$v_i(y) + \hat{t}_i \geq v_i(x) + t_i$$

for all i with at least one strict inequality. Summing over i

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$$\sum_i v_i(y) + \sum_i \hat{t}_i > \sum_{i=1}^{n} v_i(x) + \sum_i t_i$$

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which would mean that x is not utilitarian.