

Incentives and Game Theory

Jeffrey Ely

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Putting Utilitarianism to Work

Example

Suppose that you and your roommate are considering buying an espresso machine for your apartment. The machine costs \$50. Your willingness to pay is $v_1 = 40$. You know that your roommate has a willingness to pay v_2 but you don't know what it is. (Your values are zero when you do not purchase the machine.)

You and your roommate are choosing among the following alternatives.

- ① no machine, no monetary payments.
- ② espresso machine, any transfer scheme $t = t_1, t_2$ such that $t_1 + t_2 = -50$.

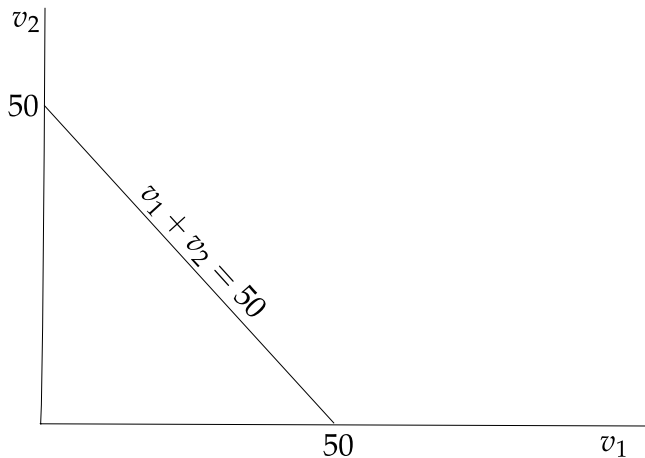
Utilitarianism in Your Apartment

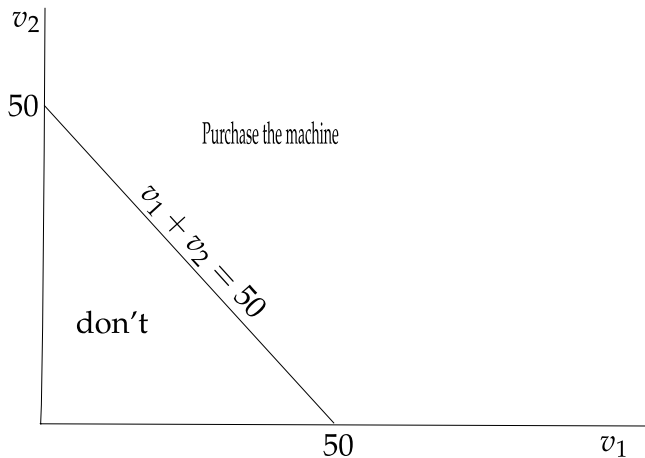
The utilitarian policy is to purchase the espresso machine if

$$v_1 + v_2 \geq 50$$

and not to purchase the machine if $v_1 + v_2 < 50$.



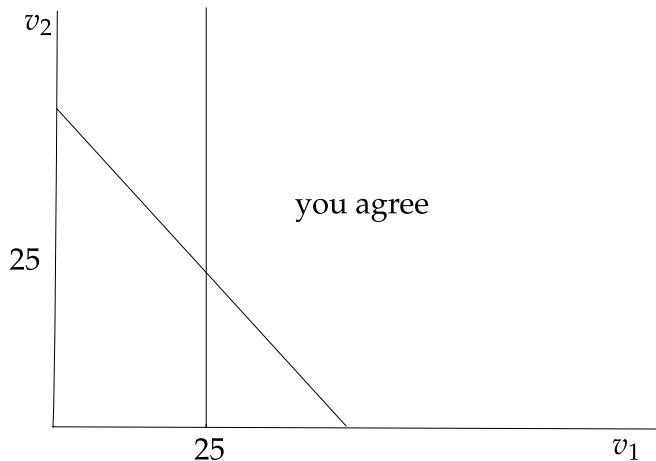




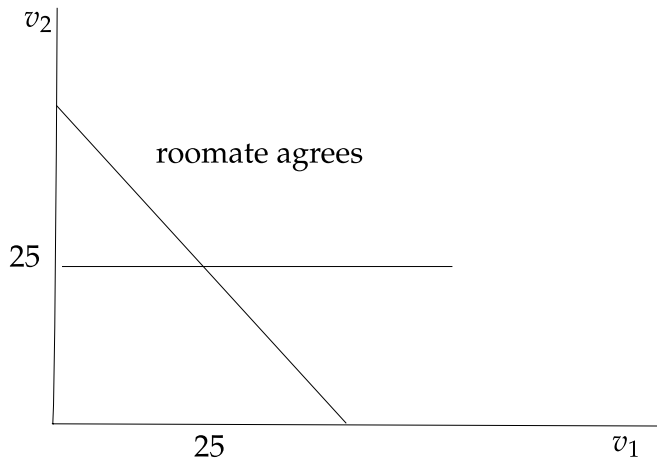
Split the Cost?

- Suppose you agree to split the cost.
- When would you be willing to do it?
- Only when $v_1 \geq 25$.
- And the same is true of your roommate.

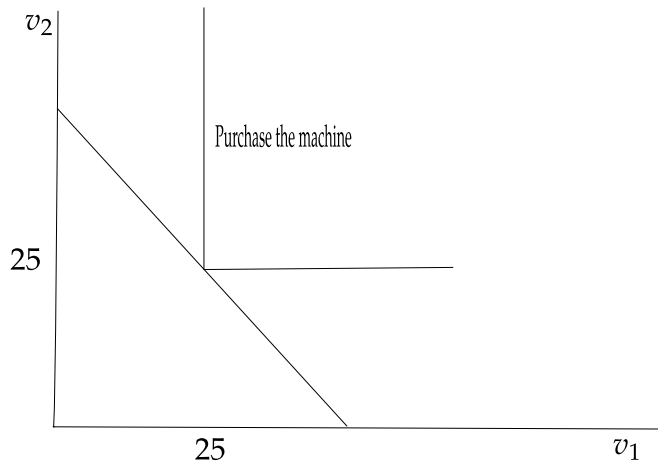
Split the Cost is Inefficient



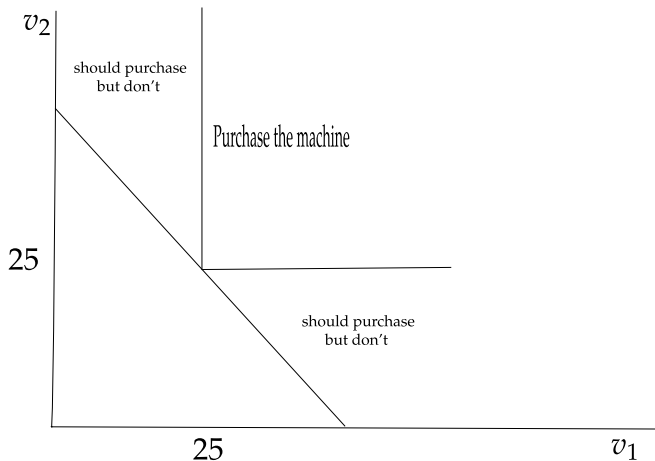
Split the Cost is Inefficient



Split the Cost is Inefficient



Split the Cost is Inefficient



Mechanism Design

Can we devise a mechanism which satisfies the following two conditions

- ① gets the two to tell truths about values
- ② allows to microwave to be bought whenever it should

Contribution Game

- The roommates simultaneously pledge a contribution (some number.)
- If the contributions add up to at least 50 then the espresso machine is bought (and the surplus divided proportionally)
- Otherwise not.

Game Theory

A game is described by

- ① The players $i = 1, \dots, n$,
- ② The choices available to each of them: A_i . (called the *actions* or *strategies*.)
 - ▶ Player i chooses one a_i from A_i .
 - ▶ The players choose simultaneously.
- ③ The *outcomes*: $a = (a_1, a_2, \dots, a_n)$ where $a_i \in A_i$ for each i .
 - ▶ When we write a_{-i} , we mean $(a_1, a_2, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$.
 - ▶ Then (a_i, a_{-i}) is another way of writing a .
- ④ Utilities: $\pi_i(a)$ (also called *payoffs*).
 - ▶ $\pi_i(a)$ is a number representing the preference for outcome a .
 - ▶ Player i wants to choose his action a_i to maximize $\pi(a_i, a_{-i})$.
 - ▶ But player i has no control over a_{-i} .

Golden Balls

Yes indeed. Golden Balls

Payoff Matrix

	split	steal
split	50, 50	0, 100
steal	100, 0	0, 0

Dominant Strategies

Definition

Strategy a_i dominates strategy a'_i if a_i always gives a payoff at least as high as a'_i and sometimes strictly higher. In formal terms, a_i dominates a'_i if

- ① for every action profile a_{-i} of the opponents,

$$\pi_i(a_i, a_{-i}) \geq \pi_i(a'_i, a_{-i})$$

- ② and for at least one action profile \hat{a}_{-i} of the opponents,

$$\pi_i(a_i, \hat{a}_{-i}) > \pi_i(a'_i, \hat{a}_{-i})$$

Super Golden Balls

- Each player has 100 pairs of balls.
- The game goes for up to 100 rounds.
- Each round an additional £100,000 at stake.
- Each time they both say `split`, they earn £50,000.
- The first round in which either of them say `steal`,
 - ▶ That person gets the whole £100,000 from that round
 - ▶ And the game ends.
- If neither chooses `steal` for 100 rounds the game ends. (And by then they have won £5,000,000.)

Iterative Removal of Dominated Strategies

- We begin by removing from consideration the dominated strategies.
- We consider the reduced game that remains.
- Now we remove strategies that are dominated in the reduced game.
- We continue with this until there is nothing more to remove.

Generous Balls

	split	steal
split	50,50	0,100
steal	0,0	1,0

The Twitter Game

As of right now I have 35 followers on Twitter. I am still feeling very lonely, so I need to get that number up to 150. Let's play a game.

- On Friday night at 11:59PM, I will check how many Twitter followers I have.
- If I have fewer than 150, then *all* of my Twitter followers in the class will have a chance to win some money. I will select one of my followers at random by a fair lottery. I will pay that person some money calculated as follows. If you were the n th person to sign up to follow me and you win the lottery then I will pay you

$$10 + (151 - n)/10$$

dollars. For example, this means that the first person to follow me (xwiz89, my most loyal follower) would win \$25. The 37th person to follow me (that could be you!, sign up now by clicking [here](#)) would win \$21.40. The later you sign up the lower your prize would be. But even the 149th person would win more than \$10.

The Twitter Game

- If, on the other hand, I make it to 150 followers, then *only* the 150th will win money, and I will pay the 150th follower exactly 1 dollar.
- (I will end the game if and when I reach 150 at any point in time before Friday April 10 at 11:59PM.) If you are the 150th follower, email me (and/or @jeffely me) immediately to claim your prize.